THE WAVE-CINEMATIC APPROACH TO THE MATHEMATICAL MODELING OF WATER FLOW IN OPEN RIVERBED

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Abstract: A macroscopic model of water flow (transfer) along a meandering riverbed with discontinuous and continuous sloping influx of downpour waters has been constructed. The resulted riverbed water discharge function is the solution for the heterogeneous equation of the telegraphic type having variable (in time) coefficients. The physical analysis of the solutions showed that, having the minimum empirical data, the proposed model allows us to carry out an estimation of the basic parameters of the movement functions of the wave water discharge along the riverbed with different flow routines and distances.

Die Welleberichtigung zur mathematischen Wasserbewegung im geöffneten Flussbett.

In der Arbeit ist das makroskopische Wasserbewegungsmuster längs der Flussbettesbiegung während des regelmässigen Regengusszulaufes gezeigt. Das bekommende Ergebnis der Flussbetthypographieursache erscheint häufig ans Militärfunktypniveau mit Koordonatenveränderungen und Zeitkoefizientveränderungen.

During the last 10-15 years, the interest for problems connected to environmental protection, rational utilization of natural resources, as well as protection against catastrophic floods has suddenly increased in many countries. This is because the scale of the dangerous consequences of the society's irrational activities and the global, inauspicious changes in the noosphere has been realized. One of these problems, the meeting point of hydrodynamics, wave theory, mass heat transfer, hydraulics, hydrology, meteorology, and other fundamental and applied sciences, is research of floods caused by catastrophic downpour freshets. Solving this hydrological problem is very important to the national economy, as very often the floods cause huge damages and human losses.

A macroscopic model of water flow (transfer) along a meandering riverbed with discontinuous and continuous sloping influx of downpour waters has been constructed. During its construction, there have been used only fundamental concepts inherent to different wave processes, and, first of all, characterizing the kinematics of the process. These are: the (phase or group) wave speed, the *extinction coefficient* (irreversible losses), the delay, the wave function (solving a telegraphic type equalization), and the radial expansion along the discontinuous and continuous specter of water influx into the riverbed. The minimum of basic notions from hydrology, hydraulics and hydromechanics are applied: stationary (installed) and non-stationary (impulse) flow routines, riverbed section area, water consumption, sloping influx, etc.

At the basis of this model lays the fundamental, and at the same time, the simplest principle, that in proximity applies to different wave processes: if, in point $\vec{R} = \overrightarrow{R_0}$, the wave function describing the substance movement is $\psi_0(t)$, then in point $\vec{R} = \overrightarrow{R_1}$ this function is as follows: , where $\psi_1(t) \approx \psi_0(t-\tau_1)\ e^{-\Theta_1}$, $\tau_1 = \int_{l_0}^{l} \frac{dz}{V(z,t)}$, $\Theta_1 = \int_{l_0}^{l} \alpha(l,t)dl;$; $V(l,t)$ is the motion speed, $\alpha(l,t)$ – the differential coefficient of wave extinction in consequence of losses, l – the length of the water influx axle along which the local wave spreading takes place (linear radial co-ordinate), $\vec{R}$ – radius vector of the observation points. The sign "≈" means that we disregard such "delicate" effects like wave field cross $\vec{r}$ ($\vec{r} \perp \vec{l}$),- diffusion, wave profile distortion as a consequence of phase speed dispersion of elementary Fourie-signal, as well as non-linear diffusion (Eilerov whirlwind type of the wave profile).

Utilization of wave-cinematic principle allows to approximately conceive the change function of water consumption in the riverbed influx as a linear superposition of discontinuous and continuous components, conditioned by the local influx of downpour waters into the riverbed system of the reservoir, both in stationary and in un-established routines.

The discontinuous consumption component $Q_d(l,t)$ can be presented by a ultimate radial row composed of N+1 wave functions; it was thorough enough studied earlier (here $Q \approx V_0 \cdot S,\ V_0$ being the medium speed of water movement in the riverbed of the river, S – real section, *l* – the length of the middle curve of the riverbed, *t* - time). A more complex problem is calculation of the continuous consumption component Q_c (*l,t*), more complex than calculation of Q_d (*l,t*). This is related to the fact that Q_c (*l,t*) is presented under the form of a curve (Duamel integral or potential lag), which depends on the density C (*l,t*) of the continuous (according to *l* and *t*) water influx into the riverbed flowing down from the active D area of the reservoir ($D: l \in [a,b];\ r \in [-R_2, +R_1];\ |D| \equiv mes(D) \approx (b-a)(R_1+R_2);$ R_1 and R_2 are the lengths of the right and left waterside slope in the active riverbed area). To calculate the functions $C(l,t) \sim \frac{\partial}{\partial l}[Q_c(l,r,t)]\ |_{\tau=0}\ (l \in [a,b],\ |t| \le \infty)$ the afore-mentioned wave-cinematic principle was used. It allows an approximate description of the flow-down process from the reservoir slopes into the riverbed. This was the purpose of introducing the density (intensity) function of the downpour flows (slope influx models) $d(l,r,t)$ ($\iint_D \int_{-\infty}^{+\infty} d(l,r,t)dl\ dr\ dt\ = U_0$ (м^3)) – the general slope influx capacity formed on the D area as a consequence of rainfall. Estimating $d(l,r,t) \approx d_0 f(l) g(r) h(t)$ (м/с) to find $C(l,t) \sim \frac{\partial}{\partial l}[Q_c(l,r,t)]\ |_{r=0}$ the curve formula $C(l,t) \approx d_0 f(l)\left[\int_0^{R_1} g(r')h(t-r'/u_1)dr' + \int_0^{R_2} g(-r')h(t-r'/u_2)dr'\right],$ was used, where $u_{1,2} \approx v_{1,2}\cos\theta_{1,2};\ v_{1,2}$ - are the speeds of the slope downfall of the downpour waters; $\theta_{1,2}$ are the inclinations of the slopes; d_0 – normalizing coefficient that depends on $U_0, |D|, T, \langle f \rangle, \langle g \rangle, \langle h \rangle;\ T$ – stands for the duration of the rainfall; $\langle f, g, h \rangle$ – medium function value.

Estimations of $f(l) \sim P_m(l),\ g(l) \sim P_n(r),\ h(t) \sim \exp\left(-|2t/T|^q\right)\ (m,n = \overline{0,4},\ q = \overline{2,10})$ allowed, with the help of the computer, a qualitative and quantitative research of the formation processes and water discharge impulse structure $Q_c(l,t)$, conditioned by continuous water influx into the riverbed circuit. The resulted riverbed water discharge function is the solution for the heterogeneous equation of the telegraphic type having variable (in time) coefficients. The physical analysis of the solutions showed that, having the minimum empirical data, the proposed model allows us to carry out an estimation of the basic parameters of the movement functions of the wave water discharge along the riverbed with different flow routines and distances. At the same time, some enlargement (according to *l*) and lengthening (according to *t*) effects of the impulse $Q_c(l,t)$, have been discovered, these being conditioned by convection processes, i.e. water transportation with different speeds. Extinction and phase-amplitude signal distortions were also studied in different hydraulic routines and riverbed flow parameters.

Notwithstanding the well-known proximity, having the minimal quantity of initial parameters, the proposed wave-cinematic approach allows us to obtain a mathematically correct, achievable and physically content-rich picture of a complex phenomenon of formation, transformation and spreading of wave water discharge in open riverbed. This structural approach can be used for elaboration of computer algorithms allowing to carry out the hydraulic calculations and the prognosis of flood formation on small and large rivers.