FRACTAL DISPERSION FRONT ON A PERCOLATION CLUSTER

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***Abstract*:** In this work the results of our percolation experiments on a porous model containing sparsely connected pores and bonds of equal size were showed. The aim is to define the fractal dimension of water flow path generated such a micro-scale percolation-like irregularity. Experimental water paths using coloured water were visualized and their fractal dimensions with the box-counting method were measured. Four regions based on more than 900 porous Hele-Shaw experiments were created. The fractal dimension of the water path backbone is ≈ 1.36.
***Keywords*:** dispersion, percolation theory, block-counting dimension.

1. Introduction

The phenomenon of hydrodynamic dispersion (the unsteady transport of a neutral tracer in a carrier fluid flowing through a subsurface regime) has been widely investigated in the fields of civil and chemical engineering [Saffman, 1959]. One can identify different regimes of tracer dispersion according to the Péclet number (*Pe*), which is the ratio between the typical time for diffusion and the typical time for convection. In the small *Pe* regime, molecular diffusion dominates the way in which the tracer samples the flow field. In the large *Pe* regime, also called mechanical dispersion, convection effects are significant; the tracer velocity is approximately equal to the carrier fluid velocity, and molecular diffusion plays little role. The tracer samples the disordered medium by following the velocity streamlines.

The classical approach to model dispersion in porous media is to consider microscopically disordered and macroscopic isotropic and homogeneous porous materials. Under these conditions, dispersion is said to be gaussian and the phenomenon can be mathematically represented in terms of the convection-diffusion equation. This traditional formalism, which is valid for Euclidean geometry, cannot be adopted to describe the global behavior of hydrodynamic dispersion in heterogeneous systems. Specifically, in the case of percolation porous media, the breakdown of the macroscopic convection-diffusion description is a direct consequence of the self-similar characteristic of the void space geometry.

In this article we discuss the results of a Hele-Shaw percolation experiment. Consider, e.g., fluid flow in percolation clusters near the percolation threshold---a model system relevant to a porous medium with stagnant small-velocity zones that are linked with large-velocity zones. In this case the typical time for convection is without bound since the velocity can be arbitrarily small in some fluid elements of the void space. Saffman showed that the mean square duration of a tracer step is not finite but diverges logarithmically unless an upper cut-off is introduced into the typical time step [Saffman, 1959]. This upper cut-off is imposed by the mass transport mechanism of molecular diffusion. The final aim is to define the fractal (block-counting) dimension of water flow path generated such a micro-scale percolation-like irregularity. Experimental water paths were visualized with coloured water and their fractal dimensions were measured with the box-counting method. The experimental results based on more than 900 porous Hele-Shaw experiments showed that the water path develops a fractal pattern. The fractal dimension of the water path backbone is ≈ 1.36.

2. Percolation experiments

The porous medium is composed of blocks of impermeable material that occupy, with a given probability, a square lattice. We consider a lattice at the site percolation threshold, so an incipient spanning cluster is formed that connects the two ends of the lattice. Previous studies modeled the convective local “bias" for the movement of the neutral tracer in the porous media assuming Stokes flow [Sahimi, 1995]. Even at macroscopically small Reynolds conditions, this assumption might be violated in real flow through porous media, specially in the case of heterogeneous materials (e.g., percolation-like structures) where a broad distribution of pore sizes can lead to a broad distribution of local fluxes. As a consequence, inertial effects might be locally relevant.

We start by describing the disordered medium and the velocity field. Our basic model of a porous medium is a percolation model [Oxaal, 1990], modified to introduce correlation among the occupancy units. We assume the existence of correlation because we obtain a better mathematical representation of transport properties---such as sandstone permeability---by assuming the presence of long-range correlation in the permeability fluctuations of the porous rock [Makse, 1996]. The permeability of rocks such as sandstone can fluctuate over short distances, and these fluctuations significantly affect any fluid flow through the rock. Previous models assumed that these fluctuations were random and without short-range correlations. However, permeability is not the result of a simple random process. Geologic processes, such as sand deposition by moving water or wind, impose their own kind of correlation.

For the determination of the hydrodynamical dispersion for the subsurface environment the error function method of solving the convection-diffusion equation can be applied. From this the so called breakthrough curve can be derived, in lab environment. This solution cannot be used in our experimental situation for the case of linear porous Hele-Shaw cell because there exits a radial increasing velocity field. Because of this effect the dispersion parameter for the radial direction is decreasing. In the radial flow the distance between the streamlines is increasing with the radii, what could be followed investigation of the increasing of the dispersion. In the interest for the determination of the concentration profile for the porous injecting system we put a gaussian error function on the front. In possession of the profile (the so called breakthrough curve) we are able to calculate the dispersion coefficient. To find a fractal characteristic of the system using the lab results the block-counting dimension was used. The number of filling circle is needed to overlay the pattern

$$N(r) \equiv r^{-D} \tag{1}$$

where *D* is the non-trivial exponent, the block-counting dimension of the pattern.

For different cases we received different dimensions changing the porosity and the pressure. That is despite of the universality of dimension as stated Maloy et al. [1988] we found powerful oscillation of the dimension in the interval [1.21,1.48].

The mask for the percolation model is generated by a computer program based on Monte Carlo experiment in possession of the grain-size distribution as a probability function. Pores were placed on the lattice sites in a 659x659 square lattice with probability p=0.59. Flow was to be allowed between nearest neighbor pores so these were connected with identical bonds. The pores and the bonds which belonged to the cluster spanning the whole lattice created the porous model medium. All pores and bonds not belonging to the infinite cluster were removed from the mask. (See Figure 1 for realization of the percolation cluster and the backbone.)

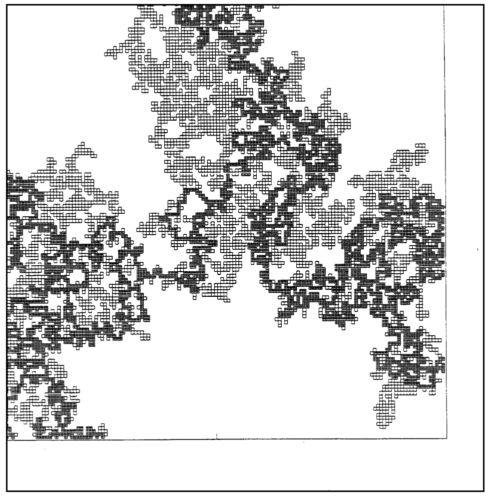

Figure 1 - Percolation cluster (nigrosin in glycerol with water) [Oxaal, 1990]

The first step in the procedure for creating the physical model was the sticking on the glass sheet epoxy based cubic particles on the position where the computer generated mask showed. The dimensions of the model were 41.3 x 41.3 cm with lattice constant 1.8 mm. Channel depth between the stuck spheres was 0.9 mm. The cubic diameter 1.7 mm and bond width 0.9 mm. The point of the injection was on a cluster pore at the geometrical center of the model.

3. Results

In the experiments we used potassium-permanganate with different concentrations. We did more than 900 experiments. We used twenty-four main types of models with discharge interval [0.01, 0.1] ml/hour. Figure 2 shows a typical lab picture with 0.05 ml/hour. It can be clearly seen the sparsely connected pore volume and the emptying of large stagnation zone area. It demonstrates the broad time distribution of potassium-permanganate particles starting out together in the center at t=0, travelling out to the edge of the model. The long vein of the clear fluid phase tells us, that first order tracer particles still have a long way to go before the edge is reached. Tracer particle diffusion into dead ends of the model makes another contribution to the time distribution.

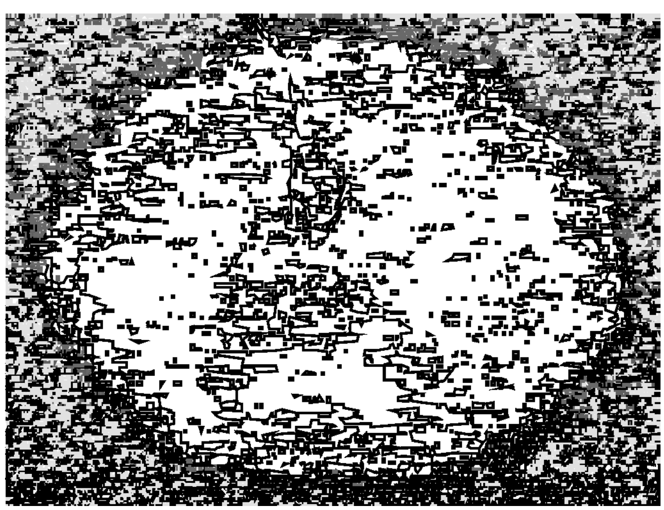

Figure 2 –Experimental picture (water with potassium-permanganate)

We noticed a broad distribution of the flow velocities in the model:

1. In dead ends there is no convective transport. Transport of tracer particles happen by molecular diffusion. (Pe=0).
2. In stagnant zones of the backbone (where a pore area is connected in two junctions to the rest of the backbone, and where the pressure drop over the two junctions is very small) diffusion will dominate. (0<*Pe*<5)
3. In other slightly faster corners of the backbone convection will dominate the dispersion process, but diffusion is not negligible. (10<*Pe*<100).
4. In the part of backbone that is a direct way out we have negligible diffusion effects. Dispersion can be described by the spreading only due the convection. (*Pe* >100).

For length scales less than the percolation correlation length (which is equal to the size of the whole model when we are at the percolation threshold) the clusters and its backbone are fractal objects. The fractal dimension of the backbone is ≈ 1.36. This value does not agree with theoretical calculations of the backbone of the percolation clusters on larger lattice, because the displacement not even fill the whole backbone due to trapped regions of porous media.

To make a quantitative description of the dispersion front we have digitized the pictures of the percolation experiments. The digitized resolution corresponded to one lattice constant per 20 pixel. We have as an approximation assumed that the gray level of the pores and bonds is a good representation of the actual concentration profile, which again is correlated to the distribution of mass. Since independent measurements have shown an experimental dependency of light intensity on the concentration of colored water and photographic film is known to have logarithmic response to light intensity we assume, that the two effects cancel and that the gray level in the photographs adequately represents the concentration. We want to see, how the cumulative mass develops as a function of radius

$$M(r \leq R) \propto R^D \quad (2)$$

and we express the mass *M* as the gray level in the digitized picture sum up for all radii *r* less than the radius *R* measured in pixels. (Figure. 3.)

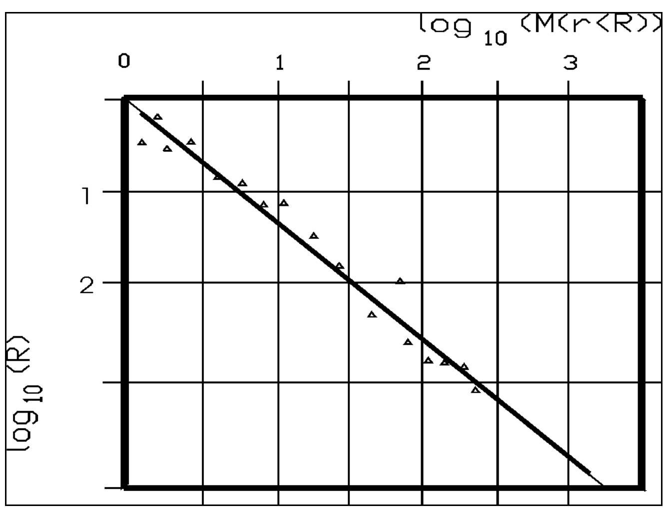

Figure 3 – Block-counting dimension of the water path

4. Acknowledgement

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