METHODOLOGY OF THE STATIC AND DYNAMIC BALANCE OF WATER RESOURCES AND NEEDS (BWRN)

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Abstract: The (summarizing) BWRN refers either to a fixed ("frozen-in") development level ("static BWRN") or allows for certain -- predicted or pre-planned – trend-like changes in the time series of the balance elements ("dynamic BWRN"). The paper describes the mathematical structure of both kinds of the BWRN, summarizing the formulae to be used for computing the final results of the balance, namely the expected values of water shortage (defined einther as the relative duration of water shortage or as the relative value of water deficit) for both kinds of the BWRN. If such an index of water shortage is lower than its prescribed upper limit value, called water shortage tolerance, the BWRN is considered active, i.e., the water management situation characterized by it is acceptable.
Keywords:

METHODIK DER STATISCHEN UND DYNAMISCHEN BILANZ DES WASSERVORRATS UND –BEDARFS (BWRN)

Kurzfassung: Die (summierende) BWRN bezieht sich entweder auf ein festgelegtes ("eingefrorenes") Entwicklungsniveau ("statische BWRN") oder ermöglicht auch die Berücksichtigung gewisser – vorausgesagter oder –geplanter – trendartiger Veränderungen in den Zeitreihen der Bilanzelemente ("dynamische BWRN"). Der vorliegende Beitrag beschreibt die mathematische Struktur beider Bilanzarten, indem er die Formeln mitteilt, welche bei der Ermittlung des Endresultats der Bilanz, nämlich der Erwartungswerte der Wassereinschränkung (definiert entweder als die relative Zeitdauer der Einschränkung oder als das relative Maβ der Fehlwassermenge) für beide Bilanzarten verwendet werden. Wenn eine solche Kennzahl der Wassereinschränkung ihren vorgeschriebenen oberen Grenzwert, die sog. Fehlwassertiűoleranz, nicht überschreitet, wird die betreffende BWRN als aktiv, d.h. die durch sie charakterisierte wasserwirtschaftliche Situation als akzeptabel betrachtet.
Schlüsselworte:

The BWRN as a basic tool of water resources planning

Water resources planning is a decision-making process by which selection is made from a set of water management measures to reach optimum solution from the viewpoint of a given set of objectives.

Generally speaking, there are two ways to make this decision:

(a) A sufficient number of project alternatives aiming to satisfy a given set of objectives must be prepared. From these project alternatives, the most feasible is selected.

(b) The components of the expected results of a planned interference are expressed as functions of parameter-combinations characterizing this particular interference. The parameter-combination (that is, the project alternative) to be realized will be selected by optimising the function upon the given set of objectives.

Solutions (a) and (b) do not differ in principle, but rather from a practical point of view. Obviously, solution (a) is simpler, but it implies several non-quantified, preliminary decisions based on experience (such as selecting in advance a "great enough" number of project alternatives to go upon). Certainly, solution (b) is theoretically more exact; but when adopting it in practice one might be compelled to introduce even such simplifications (due to lack of information and to constraints in computer technique), that this more sophisticated solution might appear unjustified.

Basically, there are two tools for solving the task of selection implied in long-term planning of water resources:

Water resources systems analysis (WRSA)

Balance of water resources and needs (BWRN)

WRSA was used successfully – whenever the conditions of application were secured – for both (a) and (b) approaches. It was adapted to practical use all over the world during the last 20-30 years (Hall & Dracup, 1970).

BWRNs are used only for the selection of project alternatives according to (a), in the case of simple – or simplified – tasks. Since 1950, several European countries (such as Czechoslovakia, the Democratic Republic of Germany, Hungary, Poland and Romania) adopted several BWRNs for checking and justification of the reality of water resources development concepts (Domokos, 1972). Several international institutions tried to form a practical, uniform methodology for BWRNs (CMEA, 1970; UN, 1974).

A BWRN refers either to a fixed ("frozen-in") development level ("static BWRN") or allows for certain – predicted or pre-planned – trend-like changes in the time series of the balance elements ("dynamic BWRN"). It is to say, the static BWRN investigates the hypothetical status when the expectation values of all balance elements are constant (allowing of course for periodic and stochastic fluctuations around these expectation values) while in the dynamic BWRN the development level can change in time, i.e. any element may have also a trend component.

Obviously, the dynamic BWRN is a more realistic description of reality than the static one. All the same, so far almost exclusively static BWRNs have been used in practice, due to their simplicity. A dynamic BWRN can approximatively be substituted by a series of static BWRNs, each of them referring to a subsequent (hypothetically frozen-in) development level.

For both the static and the dynamic BWRN, the most simplified – hence most frequently used – version is the summarizing BWRN, in which both utilizable water resources and water demand arising in the investigated area (e.g., a catchment area) are represented by resultant time functions.

Methodology of the static summarizing BWRN

The compilation of the summarizing BWRN of a given area is based on the following conditions:

(a) A long enough reach of the homogeneous time functions R(t) of resultant utilizable water resources as well as of the resultant water demand time function D(t) are known, both of them referring to the frozen-in development level. Both time functions R(t) and D(t) are supposed to characterize the whole area.

(b) The most suitable measure γ of water restriction (that is, the measure most closely related with economic losses caused by the restriction) is known.

(c) The γ^* limiting value of this water restriction index (the so-called water shortage tolerance) is also known. A water resources situation checked by the balance can be qualified as acceptable if this index is not surpassed by the actual (expected) value of γ.

If these three conditions are satisfied, the BWRN can be prepared in the following way:

(i) From the time functions under (a) the actual or expected value of the water restriction index γ discussed in (b), characterizing a given reference period T, is calculated:

$$\gamma_T = \gamma\left(R(t), D(t), T\right) \tag{1}$$

(ii) Index γ_T is compared with water shortage tolerance γ_T^* as mentioned under (c), characterizing the same T reference period. If he condition

$$\gamma_T \leq \gamma_T^* \tag{2}$$

is fulfilled for each possible T period – and the difference $\left(\gamma_T^* - \gamma_T\right)$ is not too big – the project alternative is acceptable ("the BWRN is favourable"). If the opposite case the project alternative is not accepted.

As it has been shown (Domokos, 1974), the following two restriction indices are useful in practical decision-making:

(a) The relative duration of water shortage:

$$\gamma_1 = \frac{1}{T} \int_{R(t)<D(t)} \quad t\,\varepsilon\,T \tag{3}$$

(b) The relative value of missing water amounts:

$$\gamma_2 = \frac{1}{\int_T D\,(t)\,dt} \int_{R(t)<D(t)} [D(t) - R(t)]\,dt \quad t\,\varepsilon\,T \tag{4}$$

Instead of index γ_2, the following γ_2' index is used in certain cases:

$$\gamma_2' = \frac{1}{T} \int_{R(t)<D(t)} [1 - \frac{R(t)}{D(t)}]\,dt \quad t\,\varepsilon\,T \tag{5}$$

From indices γ_1 and γ_2, the latter is more suitable and useful for most water users, exhibiting a closer relationship to real economic losses caused by water restriction. However, for reservoir design in Hungary and also in other countries the less expressive index γ_1 and related indices have been preferred, due mainly to their easier handling.

By choosing a suitable reference period T one can show that the water resources time function R(t) is an ergodic stationary stochastic process. In this case, R(t) can be substituted by the water resources distribution function:

$$F\,(x) = P\,(R\,(t) < x) \quad t\,\varepsilon\,T \tag{6}$$

where P is probability. In certain cases, by careful choice of T, one can even show that the stretch of D(t) upon T can be substituted – as a practical approximation – by a constant D value. It can be proved (Domokos, 1974), that if the distribution function F(x), and the constant D are known, water shortage indices γ_1 and γ_2 or γ_2' can be calculated by using the following simple formulae:

$$\gamma_1 = F\,(D) \tag{7}$$

and

$$\gamma_2 = \gamma_2' = \int_o^D F\,(x)\,dx \tag{8}$$

Note that equality $\gamma_2 = \gamma_2'$ is valid only in the case of D = constant.

Thus, in case D = constant, index γ_1 of equation (7) or index γ_2 of equation (8) can be compared – according to principle (ii) – with their upper limit values $(\gamma_1^*\ or\ \gamma_2^*)$. If they are smaller than these limit values, the water resources situation investigated is considered satisfactory.

Methodology of the dynamic summarizing BWRN

This methodology is necessarily a mixture of that of the static BWRN and of the simulation technique. It needs two different time scales:

- the natural, continuous time scale t, where $0 \leq t \leq T = n$ years,
- the discrete time scale (with 1 years steps) of water user development, $\tau\,(\tau = 1, 2, \ldots, \Theta)$, where Θ is the time horizon of system development.

Utilizable water resources

The time function of utilizable water resources is

$$K_\tau(t) = K_1(t) + K_{2,\tau}(t),\ 0 \le t \le T \tag{9}$$

where K_1 is the natural component (flow discharge):

$$K_1\ (t) = \bar{K}_1 + p\ (t) + s\ (t) \tag{10}$$

Here is $\bar{K}_1$ the expectation value of natural discharge (supposedly constant within Θ), p(t) its periodic and s(t) its stochastic component.

The deterministic values $\bar{K}_1$ and p(t) can be derived from the past observation series. The stochastic component can be written in the most simple case in the form

$$s\ (t) = s \cdot \xi_t \tag{11}$$

where ξ_t is a random number between 0 and 1 (whose distribution is known from the observation series) and s a constant discharge value (also known from the observations).

As for the anthropogenic component $K_{2,\tau}$ of utilizable water resources, it is supposed to depend only on the development level τ, but not on t:

$$K_{2,\tau}(t) = \sum\ k_{l,\tau} = const. \tag{12}$$

where K_i is a certain type of water resources component (e.g., water from storage reservoirs, from water transfer, etc.).

Water demand

From the plans of water resources development of the given area, the expectation value $\bar{D}_\tau$ of water demand must be known for each τ year ($\tau = 1, 2, \ldots, \Theta$). In this case, any "frozen-in" development level τ is characterized by the water demand time function

$$D_\tau\ (t) = \bar{D}_\tau + p_\tau\ (t) + s_\tau\ (t) \tag{13}$$

where p and s are again the periodic and stochastic component, to be determined by artificial data-generation.

It is an important requirement that the $s_\tau\ (t)$ component of water demand can be generated only jointly with the stochastic component s(t) of water resources (equation 11), since both components are governed by the same meteorological events.

The BWRN

For each of the development levels (years) $\tau = 1, 0, \ldots, \Theta$, the summarizing BWRN (in traditional sense) is compiled, i.e., the index characterizing the goodness of the system's operation is computed. If this index is, e.g., the relative amount of missing water γ_2 as of equation 4, then the result of the summarizing BWRN of the τ th year will be:

$$\gamma_\tau = 1 - \frac{\int_0^T \min\left(D_\tau(t), R_\tau(t)\right) dt}{\int_0^T D_\tau\ (t)\ dt} \tag{14}$$

Finally, from the series of the balance indexes $\gamma_1, \gamma_2, \ldots, \gamma_\Theta$, the critical value γ_{crit} is selected. E.g., it might be defined as the value, belonging to the abscise F = 0,9 or F = 1,0 on the distribution function

$$F\ (x) = P\ (\gamma_\tau \le x) \tag{15}$$

If the upper limit value (water shortage tolerance), prescribed on the basis of economical considerations, is γ^* and

$$\gamma_{crit} \leq \gamma^* , \tag{16}$$

then the planned development of the water resources system investigated is acceptable, otherwise it is to be rejected.

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