MATHEMATICAL MODELLING OF WATER RUNOFF DISTRIBUTION BETWEEN THE DANUBE DELTA BRANCHES

M. Kornilov[1], S. Aleshkin[2]
[1]Danube Hydrometeorological Observatory, Izmail, Ukraine
[2]Faculty of Geography, Moscow State University, Moscow, Russia

Abstract: One of the most important characteristics of the delta formation processes is water runoff distribution and connected with it the distribution of the sediments. The authors have developed a mathematical model of the Chilia delta of the Danube River. This model allows us to calculate the possible distribution of water runoff between the branches as a result of hydrotechnical measures (increase in branch and mouth bar depths, branch straightening, etc.).

1. General

The method is based on integrated modules of hydraulic resistance [2,3]. It consists in replacement of the modules of hydraulic resistance of a system of watercourses with an integrated module of resistance so that water discharge and level drop of that fictitious riverbed is the same as the total discharge and level drop of the system of watercourses. The method of integrated resistance modules is applicable to any branchy system (without cross-flow from one system to the other) when filling of the riverbed is known (at the stated discharge in the upper part of a delta or any channel network) and morphometric characteristics of all the watercourses given.

The method of integrated resistance modules helps to calculate water discharge distribution between branches in a delta and a drop of water levels between the splitting and merging points analytically, without iteration procedure.

The following equations are used for the methodology:

1) Equation of integrated module of resistance of a system of delta branches connected consequently – in parallel

$$F'_i = F_i + \frac{1}{\left(\frac{1}{\sqrt{F'_{i+1}}} + \frac{1}{\sqrt{F'_{i+2}}}\right)^2}, \qquad (1)$$

where F_i – module of resistance of entrance (main) branch of the system presented by a segment of riverbed between the neighboring knots of branching – upper and lower (Fig. 1a), F'_{i+1} и F'_{i+2} – integrated modules of resistance of the left (i+1) and right (i+2) subsystems, also calculated from the formulas of type (1);

2) Integrated module of resistance of a delta branch, flowing directly to the estuarine offshore zone (Fig. 1b)

$$F''_i = F_i + F_j, \qquad (2)$$

where F_i – module of resistance of the river arm, F_j – module of resistance of the estuarine bar. In this case the scheme of connection of the river arm and the estuarine bar is consequent;

3) Module of resistance of any river arm between the neighboring splitting points

$$F_i = \frac{l_i n_i^2}{B_i^2 h_i^{10/3}}, \qquad (3)$$

where l_i, B_i, h_i, n_i – length, the averaged values of width, average depth and coefficient of roughness of the arm;

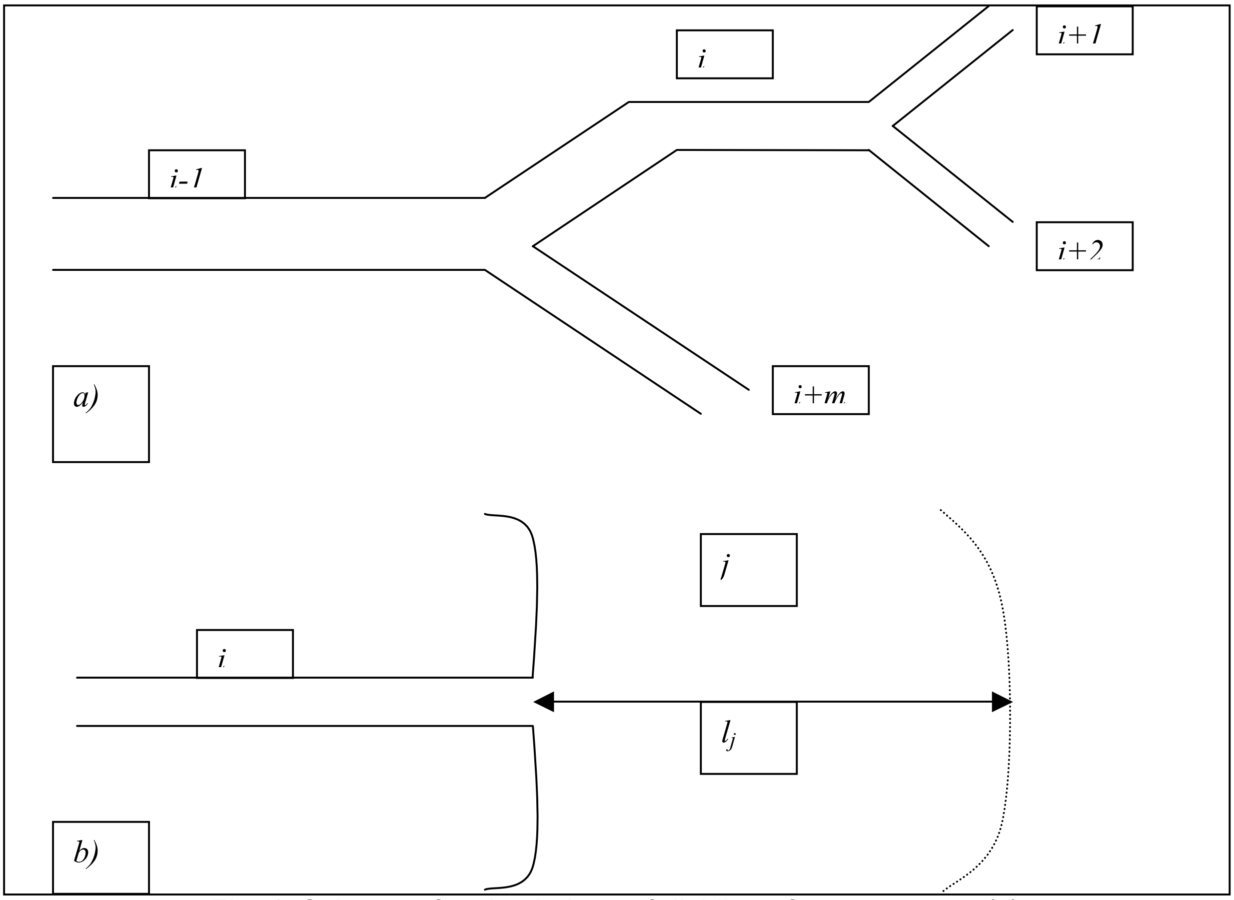

Fig. 1. Scheme of a simple knot of dividing of watercourses (a), system of watercourses and estuarine bar (b)

4) Module of resistance of estuarine bar

$$F_j = \frac{l_j n_j^2}{B_j^2 h_j^{10/3}}, \quad (4)$$

where l_i – length of estuarine bar from the estuarine section line (end of the above-water estuarine spits) to the top of a bar, B_j – half-sum of the width of riverbed at the estuarine section line and the width of the bar on its top, h_j – averaged value of the depth of the bar depression between the estuarine section line and the top of the bar, n_j – average coefficient of roughness of the bar;

5) Distribution of water discharge between the neighboring sub-systems of river arms

$$\frac{Q_i}{Q_{i+m}} = \sqrt{\frac{F'_{i+m}}{F'_i}}, \quad (5)$$

where Q_i and Q_{i+m} – water discharge from the neighboring river arms, F'_i and F'_{i+m} – integrated modules of resistance of those systems calculated from the formulas of type (1);

6) Water balance in the splitting points

$$Q_{i-1} = Q_i + Q_{i+m}. \quad (6)$$

It follows from equations (5) and (6) that at the calculated ration $\frac{Q_i}{Q_{i+m}}$ and at the known value of Q_{i-1} water discharge in the river arms heads i and i+m will be equal:

$$Q_i = \frac{Q_{i-1}}{1 + \frac{Q_i}{Q_{i+m}}}, \quad (7)$$

$$Q_{i+m} = Q_{i-1} - Q_i. \quad (8)$$

7) Drop of level between two neighboring knots will be equal to:

$$\Delta z_i = F_i Q_i^2 ; \quad (9)$$

8) Drop of level along the river arms ended with estuarine bars:

$$\Delta z''_i = F''_i Q_i^2 ; \quad (10)$$

9) Drop of level marks in the knots

$$z_k = z_m + \Delta z''_i + \sum \Delta z_i , \quad (11)$$

where z_k – level mark in the knot k, z_m- water level mark in the estuarine offshore zone, $\Delta z''_i + \sum \Delta z_i$ - the sum of level drops between the estuarine offshore zone and the knot k consisting of the drops on all the segments of the riverbed in this section.

The calculation can also be done using the formula of the type:

2. Calculated Scheme of the Danube Delta

The simplified scheme of the Danube Delta is shown on Fig. 2. The arms and their calculation sections are numbered in Roman numerals (i = I, II, III … XXXII); altogether 32 river arms and their sections are incorporated. Number zero is assigned to the Danube upstream of the delta. The splitting and merging points are numbered in Arabic numerals (k = 1, 2, 3 … 17); altogether 17 knots are incorporated. Estuarine bars are marked with capital letters (j = A, B, C … L); altogether 11 estuarine bars are incorporated.

The scheme does not include the very shallow watercourses that do not play any significant role in the distribution of the Danube water between the arms in the delta. To simplify the scheme we also do not consider cross-flows from one system into the others, which enables us to use the method of integrated modules of resistance. For example, in the reality arms XVIII and XIX join together near the place of entering the estuarine offshore zone. In the design model they are incorporated independently; it is proposed to conventionally divide the bar in the estuary of the joint arm into two equal parts D_2 and D_3.

As an example, source morphometric characteristics of the arms and estuarine bars are shown in Table 1. There the data for the most complicated and branched part of the Danube (particular delta of the Kilia Arm) are shown – the data for the river arms IX—XXX and the estuarine bars A—I under the conditions of low water period.

To calculate the modules of resistance for separate river arms (and their segments) and the estuarine bars in the Danube Delta equations (3) and (4) were used. At that, as a first approximation it was assumed that the coefficient of roughness *n* for all the river arms and estuarine bars is constant and equal to 0,023 [1].

The integrated modules of resistance for the river arms sections adjacent to estuarine bars were calculated using equations of type (2). The integrated modules of resistance of the systems and sub-systems of river arms were considered using equations of type (1). Water discharge distribution between river arms was considered using equations of type (5)—(8).

The relative distribution of water discharge between river arms is calculated by dividing the discharge in a certain arm (Q_i) by the corresponding discharge of the Danube (Q_0) or the discharge of water in the head of the studied river system or subsystem, for example the beginning of Kilijskaya Delta (Q_{VIII})- The most applicable are two kinds of relative values of water distribution between the arms of the Danube Delta - $\frac{Q_i}{Q_0} = \frac{Q_i}{Q_{VIII}}$.

Some results of calculation of discharge distribution between the watercourses in the estuary of the Kilijskoye Arm and comparison of the calculated values to the measured ones are shown in Table 2. We have to point out the satisfactory fitness of the complicated hydraulic calculation to the result of measurements. This is an evidence (with all the approximation of the calculated scheme and possible inaccuracy of source morphometric characteristics of watercourses and especially of estuarine bars) of suitability of the mathematical model for calculation of water discharge in the unstudied river arms or under or for designing.

Table 1. Morphometric characteristics of the arms and estuarine bars in the Kilia delta of Danube

Arm №	Arm name	Length L_i, m	Width B_i, m	Depth h_i, m	Estuarine bar №	Length L_J, m	Width B_J, m	Depth h_J, m
IX	Ochakovski (1)	800	458	6,2				
X	Ochakovski (2)	8000	388	3,8				
XI	Ochakovski (3)	3800	366	4,0				
XII	Prorva (1)	3000	170	4,1				
XIII	соединительный канал	1400	142	3,3	A	100	140	2,36
XIV	Prorva (2)	4000	256	2,2	B	1500	1100	2,00
XV	Potapovski (1)	900	225	5,0				
XVI	Potapovski (2)	5500	223	2,1	C	500	900	1,40
XVII	Gneushev	3700	70	3,2	D1	1200	180	2,17
XVIII	Poludenyi	7900	50	4,0	D2	1200	178	1,80
XIX	Ankudinov	11100	45	3,0	D3	1200	172	1,80
XX	S.Stambulski (1)	6900	509	7,0				
XXI	Bustriy	9300	170	7,1	E	2000	1100	3,04
XXII	S.Stambulski (2)	4980	319	6,7				
XXIII	Vostochni	7200	62	3,7	F	300	300	2,13
XXIV	S.Stambulski (3)	4600	275	6,5				
XXV	S.Stambulski (4)	3900	291	6,2				
XXVI	Limba	8240	42	2,0	I2	4000	522	1,00
XXVII	S.Stambulski (5)	500	525	3,4				
XXVIII	Kurilski	4400	26	1,5	I1	4000	518	1,24
XXIX	Tsuganski	2400	40	5,2	G	200	225	2,00
XXX	S.Stambulski (6)	2200	700	3,3	H	3000	1300	2,52

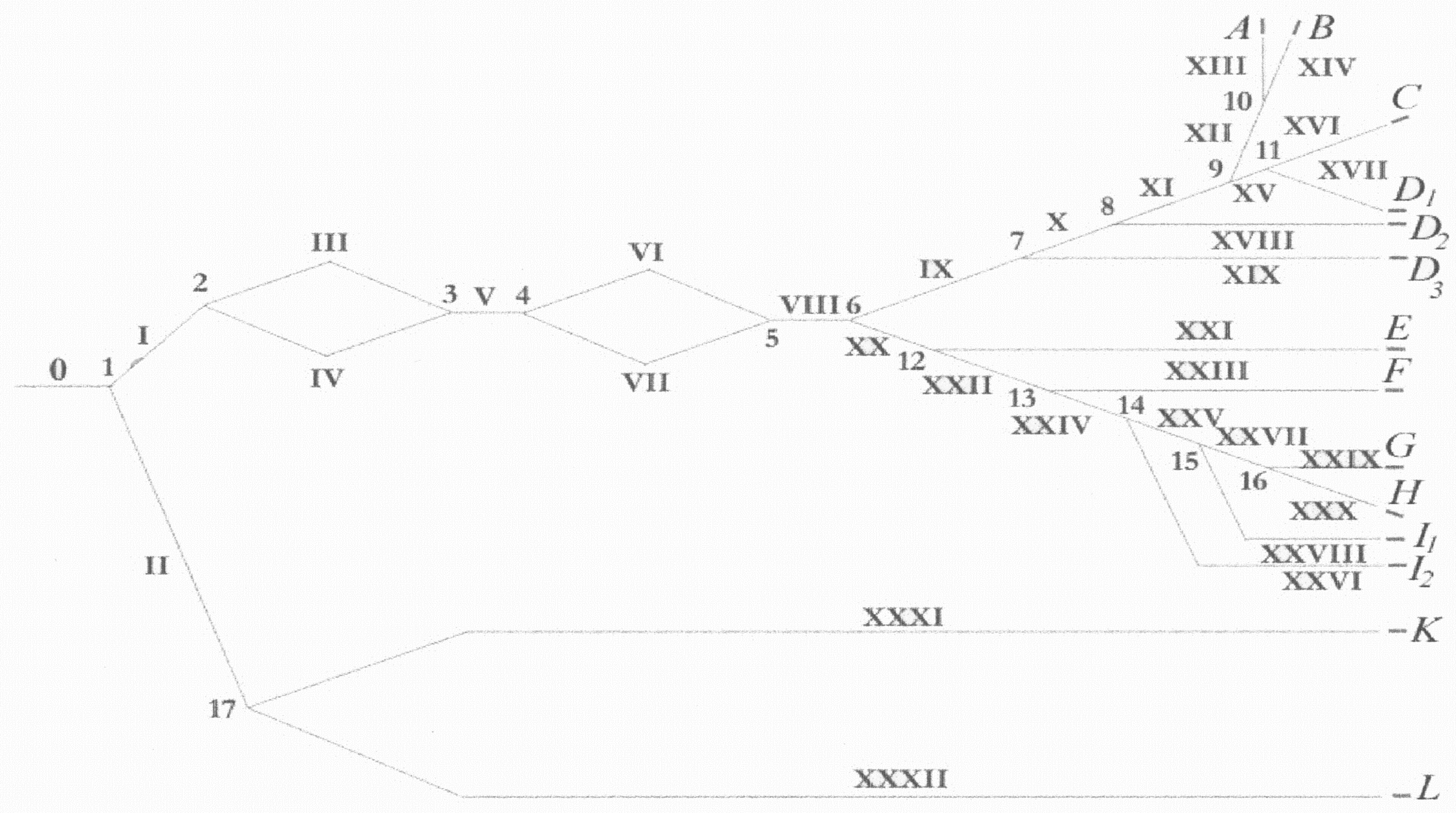

Fig. 2. Simplified scheme of the Danube Delta. I, II …XXXII – branch sectors; A, B … L – estuarine bars; 1, 2 … 17 splitting and merging points of the branches.

Table 2. Results of calculation of discharge distribution between the watercourses in the estuary of the Kilijskoye Arm

Arm №	Arm name	Water discharge, m^3/s		$(Q_c-Q_m)/Q_m$, %
		Calculated, Q_c	Measured, Q_m	
IX	Ochakovski (1)	458	-	-
X	Ochakovski (2)	419	440	-4.74
XI	Ochakovski (3)	364	-	-
XII	Prorva (1)	216	220	-1.95
XIII	Connecting channel	135	-	-
XIV	Prorva (2)	80.2	-	-
XV	Potapovski (1)	149	154	-3.57
XVI	Potapovski (2)	86.4	92.0	-6.09
XVII	Gneushev	62.1	62.0	0.17
XVIII	Poludenyi	54.9	57.0	-3.62
XIX	Ankudinov	38.7	37.0	4.51
XX	S.Stambulski (1)	1217	-	-
XXI	Bustriy	562	550	2.22
XXII	S.Stambulski (2)	655	640	2.34
XXIII	Vostochni	71.5	69.0	3.66
XXIV	S.Stambulski (3)	583	-	-
XXV	S.Stambulski (4)	570	540	5.60
XXVI	Limba	13.2	14.4	-8.34
XXVII	S.Stambulski (5)	565	-	-
XXVIII	Kurilski	5.4	5.5	-2.07
XXIX	Tsuganski	77.5	78.0	-0.69
XXX	S.Stambulski (6)	487	-	-

From the equations of types (10), (12) and (13) the values of level drop in the arms were also calculated, as well as level marks in splitting points of watercourses under low water conditions. At that, the mark of water level in the estuarine offshore zone of the Danube was taken as 0,13 m BS (Baltic System) [5]. The following ways are possible to improve the developed model and increase the precision of calculation during further use:

- specification morphometric characteristics of watercourses and estuarine bars more precisely;
- setting new values of the coefficient of roughness (instead of n = 0,023 accepted as the constant for all the watercourses), which will enable us to take into account all the errors in the calculation (simplification of the scheme of watercourses, assumption of absence of overflows between the river arms, neglecting of small watercourses), as well as local hydraulic peculiarities of the beds and estuarine bars. Selection of coefficient n for different values could help us to achieve good correspondence between the data from hydraulic calculation and measurement.

3. Example of Practical Use of the Model.

The model can be used in practice to calculate the possible changes in water regime of delta caused by both natural reasons and different hydrotechnical measures. In the first case new values should be entered into the scheme of calculation instead of current morphometric characteristics. They should be set taking into account possible natural modifications of morphometric characteristics of arms and estuarine bars as the result of wash-out, siltation, straightening of bends, extension of estuary etc. The model will also help to assess the influence of economic activities on the delta; in particular, to calculate the changes in distribution of discharge between the arms in the delta and water levels in case of artificial deepening of arms and estuarine bars, straightening or cutting the arms.

As an example of practical use of the model the possible change in discharges distribution was calculated for the system of Ochakovskiy Arm for the case of restoration of navigation through the Prorva Arm (watercourse XIV) and its estuarine bar. Transit navigation through that arm was organized in 1957 after signivicant dredging of estuarine bar of the arm. In the 90^{th} the bar silted and no serious attempt to dredge it was made because of shortage in budget. Navigation was done through the narrow and inconvenient canal from the Prorva Arm to the port Ust-Dunaisk (watercourse XIII). At present the issue of deepening of the Prorva Arm bar is being put forward again. As the preliminary calculations done with the help of the developed model have shown, if the channel through the bar is deepened down to 5.0 m (i.e. the average depth on the bar increased to reach 3.66 m) the discharge in the Prorva Arm will increase insignificantly and the discharge of the canal will decrease. In the adjacent Potapovskiy Arm the discharge of water will correspondingly decrease. If the average depth of the Prorva Arm and its estuarine bar increased to reach 3.66 m a significant change in discharge through this system will take place. The discharge of the Prorva Arm to the branching-off to the connecting canal in the low water period will decrease from 216 to 278 m^3/s or 26%, and downstream from the branching-off from 80.2 to 195 m^3/s or 138%. In its turn, the discharge through the connecting canal will decrease from 135 to 83 m^3/s (40%) and in the adjacent Potapovskiy Arm — from 149 to 121 m^3/s (20%). At that, the changes will affect even the distribution of discharge in the point 6 where the discharge from Ochakovskoye Arm will grow from 458 to 488 m^3/s (5%) and the discharge of the Starostambulskoye Arm will correspondingly decrease from 1217 to 1187 m^3/s (2%).

4. Conclusions:

The developed mathematical model of distribution of water discharge between the arms in the delta describes the current hydrology quite adequately and suits for forecasting of changes at planned hydrotechnical measures.

5. References:

Mikhailov, V. N. (1971): Current and river bed dynamic in untidal deltas. Moscow.

Mikhailov, V. N., Rogov M.M., Chistyakov A.A. (1986): River deltas. Hydro-morphological processes. Gidrometeoizdat, St.Peterburg.

Polonski, V.F., Lupatcev, Y.V. and Scriptunov, N.A. (1992): Hydrological and Morphological Processes in the river’s estuaries and method’s of their calculation (forecasting). Gidrometeoizdat, St.Peterburg.