FRACTAL MODELLING THE FRUSTRATED AQUIFER SYSTEMS: ANNUAL RUNOFF ANALYSIS

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Abstract: We consider methods for an ascertainment of fractal dimensions of some hydrological variables. Specifically, we investigate the annual runoff for the Ukrainian rivers and reveal scale invariance for distribution of this variable by using statistical parameters such as arithmetic average, coefficients of variation, skewness, and auto-correlation. It is shown that the fractal dimensions for the arithmetic average and coefficient of variations amount to 1.72 and 1.63 respectively. The coefficients of skewness and auto-correlation are related to the spatially uncorrelated variables. Temporal components of empirical orthogonal function decomposition for the annual runoff are used to reveal properties of time invariance for the annual runoff. The first components of decomposition are analyzed and its connection with factors of creation for annual runoff is investigated. It is shown that first and second components represent the large- scale atmospheric forcing of annual runoff creation. The time part of first component describes most general patterns for the annual runoff fluctuations of Ukrainian rivers. Namely this variable is subject to the fractal analysis. Here the variational function $F_2(s) \sim s^H$ is used as a property of spatial-time variation for the annual runoff (H is the exponent of scaling identical with the fractal dimension). It is determined that H = 0.77 and this agrees to the hypothesis of Hurst's universal exponent.
Key words: annual runoff, fractal dimension, empirical orthogonal functions

FRAKTALE MODELLIERUNG DER FRUSTRIERTEN WASSERFÜHRUNGENSSYSTEME: ANALYSE DES JAHRESABFLUSSES

Zusammenfassung: Es wurden die Methoden der Feststellung der fraktalen Dimensionen der hydrologischen Abmessungen betrachtet. Für die Bestimmung der Maßstabsinvarianz in der Verteilung des Jahresabflusses der Flüsse in der Ukraine wurde die Raumverteilung seiner statistischen Parameter: des arithmetischen Mittelwerts, Variationsbeiwerts, der Asymmetrie und Autokorrelation untersucht. Es wurde angezeigt, dass die fraktale Dimension des arithmetischen Mittelwerts und Variationsbeiwerts 1.72 und 1.63 (entsprechend) bilden. Der Asymetrie- und Autokorrelationskoeffizient wurden auf die Anzahl der unkorrelierten im Raum bezogen. Für die Bestimmung der Eigenschaften der temporalen Invarianz des Jahresbaflusses wurden temporale Komponenten der Zerlegung der Felder des Jahresabflusses mit empirischen Orthogonalfunktionen ausgenutzt. Es wurden die ersten Komponenten der Zerlegung und ihre Verknüpfung mit Faktoren der Bildung des Jahresabflusses untersucht. Es wurde angezeigt, dass die ersten 2 Komponenten den Einfluß der größten Luftprozesse auf die Bildung des Jahresabflusses haben. Die temporale Komponente der ersten Komponente der Zerlegung der Felder des Jahresabflusses beschreibt die allgemeinsten Gesetzmäßigkeiten der Schwankungen des Jahresabflusses der Flüsse in der Ukraine, wonach sie auf die fraktale Analyse gestellt wurde. Als Charakteristik der räumlich-zeitlichen Variation des Jahresabflusses wurde die Variationsfunktion $F_2(s) \sim s^H$ ausgenutzt, wo H ein Exponent der Maßstabsbestimmung ist, der der fraktalen Dimension identisch ist. Es wurde festgestellt, dass H = 0.77, was der Hypothese des Hurst-Exponenten entspricht.
Schlüsselworte: Jahresabflusses der Flüsse, fraktalen Dimensionen, empirischen Orthogonal funktionen

1.Introduction

A new approach to the neural networks and multi-fractal modelling the frustrated hydrological systems is developed and numerically realized. It is well known that hydrological (etc.) systems (and the dynamics of their key characteristics fluctuations) can be described as a mechanical dissipative multi-body systems, which are fundamentally non-linear (Davis, 1991; Kothyari and Singh, 1999; Loboda, 1998;). General non-linear parameter dependent dynamical dissipative systems very often have parameter ranges, in which the dynamics is chaotic. Chaotic behaviour in the sense of a fully deterministic evolution of the systems in time bounded in phase space with sensitive dependence on initial conditions, might therefore be expected to occur in above cited systems. Dissipative non-linear systems typically have a long-term behaviour, which is described by an attractor in phase space. At the same the chaotic dynamics in details is often unknown. It is well known that an attractor is called strange attractor if its dimension is non-integer, i.e. fractal. Non-linear systems of fractal objects like interfaces or time-series is their scaling property related to invariance under magnification. For uniform fractals the scaling is uniquely described by one-fractal exponents, the so-called fractal dimension. In last years studying the fractal properties of dynamical systems is of a great interest. In this paper we consider an effective method for treating the non-linear complex systems, based on the “neural networks” and multi-fractal modelling (Glushkov *et al.*, 2001; Loboda, 1998; Loboda, 2001). Approach developed allows getting a possibility of forecasting the evaluation dynamics, including the extreme phenomena in non-linear complex systems. We apply these models to treating the chaotic dynamics and fluctuations of the annual runoff for natural rivers.

2. Multi-fractal modeling of nonliniar hydrological systems: annual runoff time series and fractal dimension

In last years the fractal structures in hydrodynamics, astrophysics etc. attracts a great interest. Different theoretical models are proposed and experiments in laboratories and nature objects are carried out (Cschertzer D.,Lovejoy S.,1990). A principal question is in what degree the properties of observed fractal structures are general? Recently it has been found that the similar large scaled fractal structures (with fractal dimension *D~4/3)* may be realized in laboratory turbulence (space scale is $10\sim^{1}$ м), in an ocean and the galaxies formations (scale till 10^{2} Mps). These structures have the percolation character (Lukkin,1988).

Probably the most number of papers regarding the fractal structures is devoted to the turbulence phenomenon. Usual application of the fractal formalism here as follows (Bershadsky A.G.,1990). The region, where the turbulent liquid moves is divided on the cubic cells with the characterized Kolmogorov's scale т. Можно There is existed a critical concentration of cells with probability $0<p_c<1$ when at first unlimited cluster of turbulented cells arises that leads to radical changing in the energy transformation. Before this moment, introduced in motion region energy resulted in the increasing number of cells and dissipation. After appearance of cited cluster the cells concentration will be increase. Its appearance, as it's well known, is a critical phenomenon. Characterized size of vortex cluster *l* в(in region of p_c) (Bershadsky A.G.,1990): $l\sim|P_c-P|^{-v}$. A critical size v is an universal parameter and independent upon the space topological dimension. This parameter (typical value v~0,9) is linked with fractal dimension D_s of the vortex cluster skeleton. Further if the initial filed of large scaled velocity is known (the vortexes of scale l_o are excited) then a cascade scale dividing process leads to hierarchy of vortexes of the scales $l_n\sim q\text{-}^{n} l_o$ *(q* — the scale division parameter). Process of the energy transfer on scaled cascade is chaotic one. As result, anisotropy and large scaled inhomogenity of the velocity initial field influences on statistical pulsation regime in less degree during decreasing the scale that is led to scaled invariance and local anisotropy on sufficiently little scales (l_o»l_n»?). For isotropic pulsation the energy distribution on scales *(*$l\sim k^{1}$*, k*—wave number) is defined by spectral density *E(k).* Simple physical arguments allow to introduce a characteristic pulsation period: $T_m\sim [E(k)k^{3}]^{-1/2}$, $k_m \sim l_m^{-1}$. It may be interpreted as time for exciting the vortexes of scale l_{m+1} by the vortexes of scale *lm* . A time for exciting the full cascade of vortexes: $t_\infty \sim \sum_{m=0}^{\infty} T_m$. The more interested value is $(t_\infty - t_m) \sim \sum_{m=M}^{\infty} T_m$. Under sufficiently big values of M it is true a scaled invariance and scaling representation: $E(k) \sim k^{a}$. After M dividing fractions insist of the single initial vortex it will be N $\sim q^M$ number of vortexes of the scale $l_M \sim q^M l_0$. This system of vortexes will occupy some volume in a space with effective size l_0, and it can be written: $N \sim l_*^{D_s}$, where D_s is a fractal dimension. Further one can write the following expression: $l\ (t_M\ .) \sim (t_\infty - t_m)^{2/(a-3)D_s}$. Numeric calculations of the fractal dimension allowed to obtain the following value for dimension: 1,35±0,05. Another interesting example is the fractal especialitites in dynamics of ocean system. In fig.1 there are presented data (from ref. Oceanology) of observations in different points of thermocline at the horizon of 100m. Here there are regions with scaling (parameter «4/3» is indicated by solid lines) in the large scales part of spectra.

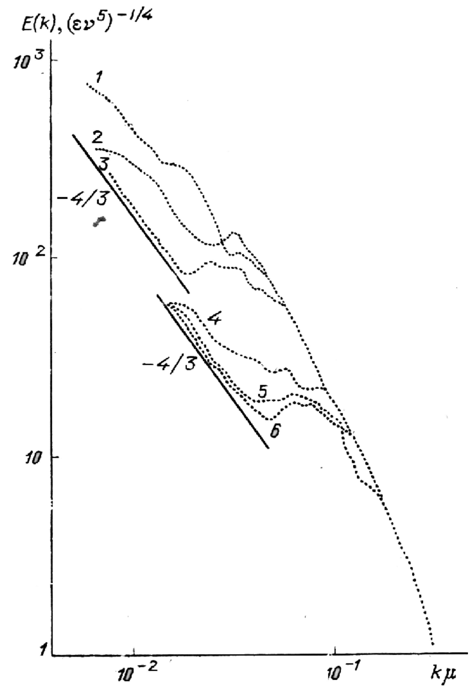

Figure 1- Fractal especialitites for in the large scales part of spectra (for ocean system: thermocline at horizon 100m)

Another examples of the fractal structures are the ones in the astrophysical objects. Under supposition that a substance in the universe is in turbulent motion, then the large scaled accumulations (galaxies and clusters) occupy the volume where an active dissipation of energy takes place, i.e. fractal dimension of the turbulent dissipation field coincides with dimension of field of the substance density in the form of galaxies and their accumulations. For these systems there is obtained the following value of the fractal dimension: 1,3±0,I , i.e. practically «4/3». In this light especial interest is called by problem of determining the fractal dimension of the other nature objects, namely, the aquifer systems. More exactly, speech is about a search of the fractal properties in the key physical characteristics of the river systems. We will discover the fractal properties in the time-series of run-off (annual run-off) for some nature aquifer systems. As an example, two rivers systems are considered.

3. Procedure for ascertainment of fractal dimensions of river systems

On an investigation of runoff variables a calculation technique for the fractal dimensions differs from the geometrical approach. Half of century ago Hurst (1951) shown that the long-range statistical dependencies appear in the series of annual runoffs for different rivers. These dependencies indicate the presence of self-similarity properties in the oscillations of runoff and hydrological processes. Per se Hurst discovered the presence of self-affinal properties in the observational series of runoff. A definition of fractal properties for hydrological variables implies statistical scale invariance (scale self-similarity). i.e. the similarity with some scale transform for any part of object to the whole one or to any part from small ones.

If time series is stationary then for the time scaling we can apply the technique of standard spectral analysis, i.e. we show a power spectrum $E(f)$ for these time series as a frequency-dependent one. For the stationary time series under long-term correlations should be the dependence as follows

$$E(f) \sim f^{-\beta}, \qquad (1)$$

where f is the frequency. Here the exponent β is inversely connected to the exponent γ of corresponding autocorrelation function $r(s)$ as

$$\gamma = 1 - \beta, \qquad (2)$$

where

$$r(s) \sim s^{-\gamma}. \qquad (3)$$

On the other hand, the exponents γ and β serve for the scale of argument. Note that if $\beta = 0$ then we may assume that data is not correlated and the process is considered as the "white noise" one.

For the case if not basic centralized series ($\varphi_i = x_i - \bar{x}$) are subject to Fourier transform but their consecutively integrated magnitudes z_n

$$z_n = \sum_{i=1}^{n} \varphi_i\,, \quad n = 1, 2, ..., N \qquad (4)$$

then according to the Wiener-Klintchin theorem the spectral density can be represented as follows

$$\tilde{E}(f) \approx f^{-2-\beta}, \qquad (5)$$

where $2+\beta$ can be also considered as the scaling index.

The scaling and the ascertainment of fractal dimensions for most hydrometeorological variables can be obtained by investigating the time or spatial variation of variable investigated in the time or space with a step s. A fluctuation of characteristics investigated on some period or given distance is implied as a variation. For the multifractal approach the integrated function performing a decomposition of set into subsets for the consequent ascertainment of scale self-similarity becomes (Koscielny-Bunde, 2003)

$$F_q(s) = \sum_{\nu=1}^{N_s} \left| z_{\nu,s} - z_{\nu-1,s} \right|^q = s^{\tau(q)}, \qquad (6)$$

where the variable q is the non-zero real value; N_s = int(N/s); ν is the number of non-overlapping segments; $z_{\nu,s}$ is the magnitude of the z_n (see Eq. (4)) calculated for each segments; and $\tau(q)$ is Rényi's scaling exponent or fractal dimension. The difference $z_{\nu,s} - z_{\nu-1,s}$ corresponds to the increment z_n on the segment s.

If $\tau(q)$ obey the linear dependence by τ then it can be assumed that the event investigated is monofractal; in the converse case this event is considered as multifractal.

Under $q = 2$ we obtain equation for the standard analysis of fluctuations; in the general case, this analysis is represented by

$$\sqrt{M(s)} = F_2(s) = \sqrt{\frac{1}{N_s}\sum_{\nu=1}^{N_s}\left(z_{\nu,s} - z_{\nu-1,s}\right)^2} \,. \quad (7)$$

For uncorrelated variables $F_2(s) \sim \sqrt{s}$. If in the time series there exists long-term correlations then we may assume that the variance $F_2(s)$ is scaled with the self-similarity exponent H

$$\sqrt{M(s)} = F_2(s) \sim s^H. \quad (8)$$

It is ascertained that the exponent H is connected with the exponents γ and β as follows

$$H = 1 - \frac{\gamma}{2} = \frac{1+\beta}{2}. \quad (9)$$

For monofractal data the exponent H corresponds to the Hurst exponent.
The function $F_2(s)$ can be considered as the square root of structure function, i.e. $F_2(s) = \sqrt{M(s)}$. This means that analyzing the hydrometeorological variables with the stationarity, which is local and remains on the relatively small intervals of argument variation, the structure function $M(s)$ can be considered along with the correlation one. This structure function is determined as the average of distribution for the squared difference of the sections of random function. If the stationary function is specified in the discrete measure points then the structure function can be represented by

$$M(s) = \frac{1}{(n-s)}\sum_{i=1}^{n-s}\left(x_i - x_{i+s}\right)^2 \,. \quad (10)$$

These structure functions can be both temporal and spatial ones. The latter represent the average of distribution for the squared difference of characteristics in the two points located on the distance ΔL one from another. Kolmogorov (1941) has developed and used the spatial structure function for the scaling of turbulent eddies and the velocity of flow was considered as the estimated characteristic.
The structure functions are assumed as a basis of the variation approach (Mark and Aronson, 1984) developed to examine the self-affine objects. The essence of this approach lies to trace the behavior of variation F for come function Z given on some set of points on a plane. F is determined by

$$F = \langle (Z_i - Z_j)^2 \rangle, \quad (11)$$

where Z_i and Z_j are the values of function in the points i and j, and the angle brackets signify the averaging in whole set of points. This corresponds to the equation of self-affine function (9). If the variation F is scaled with the self-similarity exponent H, i.e. $V \approx L^{2H}$, then the plane can be considered as self-affine one with the fractal dimension $d = E - H$, where E is the Euclidean dimension. Hence we receive results represented usually in the double logarithmic scale as the dependence $\sqrt{V}$ on the distance L.

4. Resalts

4.1. The properties of self-similarity in the spatial distribution for annual runoff's statistical parameters of Ukraine's rivers.

Now we apply Mark and Aronson's approach of variations to determine the statistical scale invariance in the spatial distribution for annual runoff's statistical parameters. As an example, we consider the spatial distribution of long-term means for the annual runoff $\bar{q}$ in the right-bank Ukraine (79 hydrological sites). The spatial variation is evaluated as the spatial structure function in Eq. (10). This function is represented by the magnitudes $\bar{q}$ for each spatial point as follows

$$M(\Delta L) = \frac{1}{m-1}\sum\left(A_i - A_j\right)^2 , \tag{12}$$

where $M(\Delta L)$ is the magnitude of spatial structure function in the segment ΔL; A_i and A_j are the characteristics in the points *i* and *j* (in our case $A_i = \bar{q}_i$); and *m* is the number of pair for magnitudes occurring in the segment ΔL.

Within the bounds of Ukraine the square net with the sides of 75 km is specified. The distances apart the centroids of basin are determined by using the false coordinates calculated by the Pythagorean theorem

$$L_{i,j} = \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2} , \tag{13}$$

where *L* is the distance apart the objects. This distance is evaluated for each pair of objects *i*, *j* by the false coordinates of object location x_i, y_i and x_j, y_j.

With the purpose to calculate the spatial structure function the nonoverlapping segments (gradations) ΔL are given. The pairs of basins falling into given gradation are chosen on basis of distance matrix. The difference $(A_i - A_j)$ is calculated for each pair of basins and the corresponding magnitude of structure function (see Eq. (10)) is estimated for each gradation. Computational results are usually represented as the diagram of $M = f(L^2)$ or of $\sqrt{M} = f(L)$. In our case, Figure 2 shows that for $L \leq 400$ km the dependence of variation on the distance has a power nature with the exponent $H = 0.28$. This exponent is determined taking the double logarithm. For $E = 2$ we have the desired value of fractal dimension ($d = 1.72$). Similarly, invariance's properties in the spatial distribution for annual runoff's coefficient of variation C_v are determined. For this coefficient the fractal dimension is 1.63. In contrast to the averaging magnitudes, the spatial correlation of C_v is observed for the distance $L \leq 200$ km only. At the same time statistical parameters such as the auto-correlation coefficient and the skewness are not actually correlated in the space. In practice this appears as short distances, on which the spatial structure function amounts to the "satiation". It is obvious that for this case possibility to determine the exponent *H* not offers.

4.2. Property of self-similarity for annual runoff series on the basis decomposition of runoff fields

The approaches stated in Section 3 are useless for the fractal analysis of non-stationary functions as these approaches can lead to the erroneous results. To exclude trends caused by the seasonal cycles, economical activity, and global warming, the approaches of the decomposition or wavelet analysis are applied. By means of these approaches the filtration of initial data is carried out.

As consistent with this method (e.g. Obied and Creuten, 1986), any matrix entry of initial variable φ_{ij} (the latter means the *i*-th objects at the *j*-th time point) can be calculated if the eigenvectors problem is solved

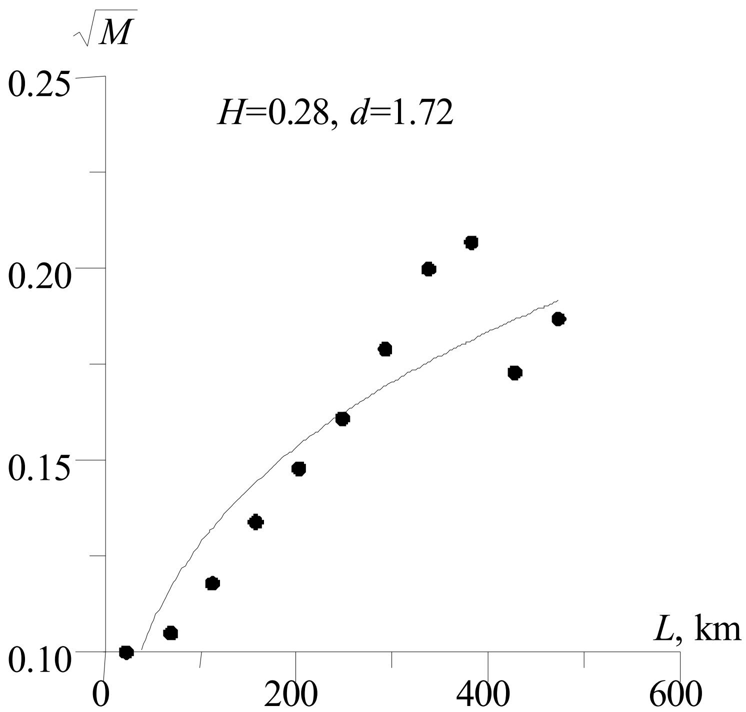

Figure 2 - The dependence of magnitudes $\sqrt{M}$

on the distances between the centroids of basins L

$$\varphi_{ij} = \sum_{k=1}^{m} U_{ki} Z_{kj} \quad \text{as} \quad i = 1,m; j = 1,n. \tag{14}$$

In Eq. (11), φ_{ij} are the components of *j*-th random vector (field) for the centralized and normalized initial data; U_{ki} are the weighting coefficients reflected the contribution of *i*-th object into each *k*-th component or, in other words, the components of eigenvector for correlation matrix; Z_{kj} are the components of *k*-th decomposition component; *m* and *n* are the number of objects and the initial series length respectively.
The magnitudes of U_{ki} vary spatially under the change of object but is time-independent. The system of function U_{ki} is often represented as the function of coordinates (x_i, y_i) for *i*-th objects as follows

$$U_{ki} = f(x_i, y_i) = U_k(x_i, y_i). \tag{15}$$

The components of row-vector for the matrix *Z* [Z_{k1} Z_{k1} .. Z_{kp} .. Z_{kn}] can be represented as the function of time (amplitude function) and are common for all objects

$$Z_{kj} = f(t) = Z_k(t). \tag{16}$$

First component of decomposition describes most part of unitial dispersion. Amplitude function of first component reflects main proreties of annual flow fluctuation, characteristic for studing region.

We analysed the amplitude functions Z_{kj}. It is determined that the amplitude function for the first component of decomposition Z_{1j} represents main patterns for the annual runoff variations in the considered region. This time distribution can be interpreted as territory-averaged variation of runoff. Such fluctuation is conditioned by global-scale processes. Therefore the scaling of first amplitude function using methods of fractal analysis allows to determine the dimensions both single hydrological object and complex of ones.

We considered longest series for the annual runoff of Ukraine's rivers such as Desna (Chernigov), Prut (Chernovcj), Severskiy Donets (Lysichansk), Prypyat' (Mozyr'), Dnestr (Galich), Yuzhniy Bug (Aleksandrovka), and Dniper (Rechitsa) with total duration of 74 years (1914-1987 years). The first amplitude function Z_{1j} (Fig. 3), which is unified for whole Ukraine and describes 48% of initial data dispersion, is subject to the analysis of variance.

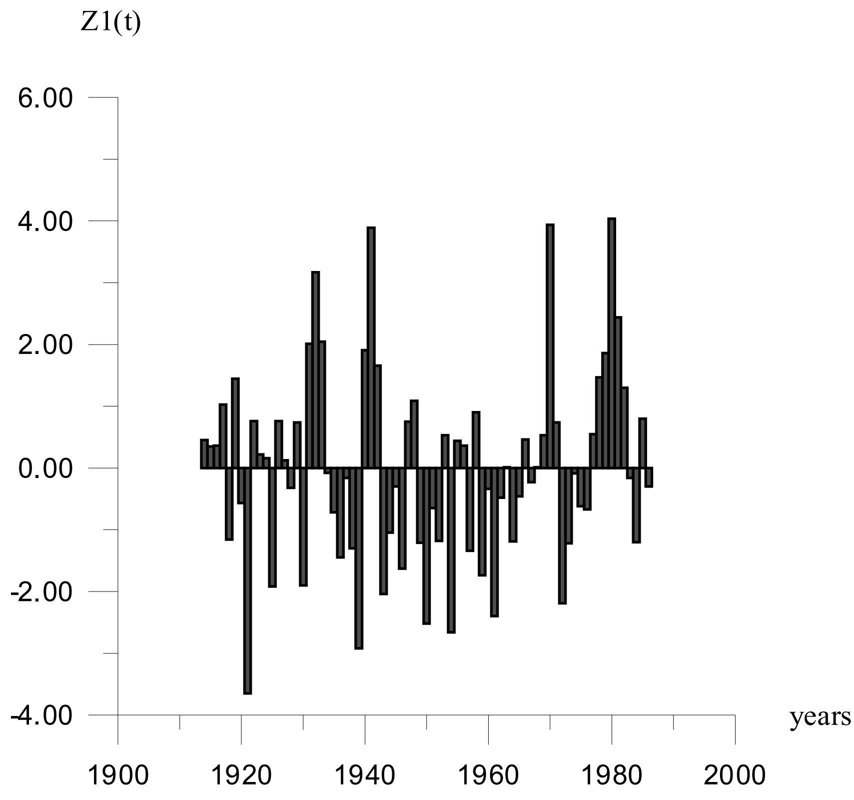

Figure 3 - First amplitude function of annual runoff of Ukraine

Variation function $F_2(s)$ calculated on first time component of decomposition Z_{1j} by using Eqs. (4) and (7) is power-like. The exponent H is determined in log-log plot. Figure 3 shows that the curve is fitted by the straight line with tangent of slope angle 0.78 at time scale up to 10 year. In accordance with Eq. (9) $\gamma = 2 - 2H = 0.46$. For $s > 10$ the function $F_2(s)$ works for the "satiation state" $H \to 0.5$. The latter indicates the deficiency of correlation.

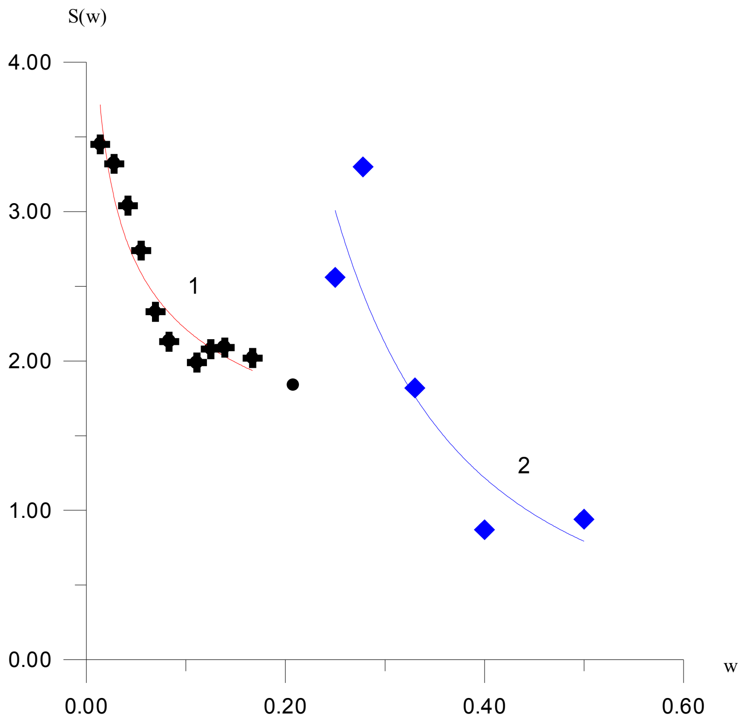

Figure 4 – Spectral densiity for first (1) and second (2) amplitude functions

5. Conclusion

Approach developed allows getting a possibility of forecasting the evaluation dynamics, including the extreme phenomena in non-linear complex systems. We apply these models to treating the chaotic dynamics and fluctuations of the annual run off for natural rivers. Within neural networks-like model we introduce a non-linear component representing the immediate and moderately retarding response and linear component representing the retarding response of the complex system. The output function Z within systems model is as follows (Kothyari and Singh, 1999):

$$Z_t = \sum_{j=1}^{J}\sum_{i=1}^{n(j)}\sum_{k=i}^{n(j)} U_{i,k}^{(j)} P_{t-i+1}^{(j)} P_{t-k+1}^{(j)} + \sum_{j=1}^{J}\sum_{i=1}^{k(j)} U_{i+n}^{(j)} P_{t-(i+n)+1}^{(j)} , \qquad (17)$$

where $j = 1,2,...,J$ is the number of independent inputs, J is the number of subsystems, $(n+1)$ is the total memory length of model; P is the matrix of the j-th input series, corresponding to the j-th sub-system; $U_{i,k}$ – the ordinates of the non-linear part of the response function, U_i – the ordinates of its linear part. The solution of model master equation for a calibration series of N outputs values $Z_1,...,Z_N$ can be written in vector-matrix form as:

$$Z = P^{(1)} U^{(1)} + \ldots + P^{(N)} U^{(N)}. \qquad (18)$$

In contradistinction to standard models, model proposed allows to account the essential non-linearity of processes, inverse links, minimally realized governing elements.

Non-uniform and multi-fractal objects can be more completely characterized by spectrum of $D(q)$ fractal exponents, where q is a real number, the so-called generalized dimension, where the fractal dimen-sion is equal to $D(0)$ and the function $D(q)$ is generally referred to as multifractal spectrum (Davis, 1991).

Mathematically, the general aim of the multifractal formalism is to determinate the $f(\alpha)$ singularity and our analysis (annual runoff fluctuations) shows that the average fractal dimensionality is 1.3-1.9.

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