10 THE INVESTIGATION OF RIVERBED EROSION IN THE MOUNTAINOUS RIVER. APPLICATION TO RABA RIVER

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Abstract: Erosion which occurs in mountain rivers is observed in the catchment areas and in the riverbeds. In the paper the investigation is carried out for the riverbed erosion.
The aim of the paper is to establish the main trends of changes at the gauging station caused by riverbed erosion in the Carpathian rivers, observed in a long time period 100 years). The process of riverbed erosion can be considered as a stochastic process, hence the investigation of the intensity of riverbed erosion presented in this paper is based on the method of statistical analysis. The consideration is based on the minimal annual water stages. The assumption is that the minimal annual water stages correspond with the changes of the riverbed level.
The computations has been carried out for Gdów and Proszówki gauging stations on the Raba River. There is also applied the method of establishing a forecast of the riverbed erosion.
Keywords: mountainous river, riverbed erosion

EINE UNTERSUCHUNG DER FLUSSBETTEROSION IN EINEM GEBIRGSFLUSS: HIER DEN FLUSS RABA BETREFFEND

Zusammenfassung: Gebirgsflüsse und -bächer charakterisiert eine sehr hohe Erosionsintensität, die sowohl das Flussgebiet als auch das Strombett betrifft. Diese Besprechung beschränkt sich auf die Analyse des Strombetterosionsprozesses.
Das Thema der Besprechung ist die Bestimmung der erosionsbedingten Veränderung des Flussbettes eines Karpatenflusses innerhalb einer längeren Zeitperiode und eine Ursachenanalyse der wesentlichen, plötzlichen Senkung des Strombettes.
Der Verlauf des Strombetterosionsprozesses wurde beurteilt anhand der Jahresmessungen der niedrigsten Stände der Flusswasserstandquerschnitte. Die Überlegungen basieren auf dem statistischen Zurücktretensmodell unter der Annahme, dass die Tendenz der Jahresniedrigwasserstände während der Jahre (100-Jahre-Periode umfassend) dem Bild des Strombettes in eben diesen Jahren entspricht.
In dieser Bearbeitung wird die Intensitätsanalyse des Strombetterosionsprozesses im Fluss Raba - Querschnitte auf der Höhe von Gdów und Proszówki - vorgestellt. Es wurde auch eine Stromflusserosionsprognose anhand des Modells, das für die Analyse der Wasserstandsprofile des untersuchten Strombettes erstellt wurde, ausgearbeitet.
Schlüsselworte: gebirgsfluss, flussbettererosion

1. Introduction

Erosion which occurs in mountain rivers is observed in the catchment areas and in the riverbeds. All natural forces (wind, changes of temperature, precipitation) lead to sediment movement compatible with the gravity in order to level the terrain. The products of erosion, which occurs in the catchment area, finally enter the streams and rivers. Depending on the hydrodynamic conditions in the riverbed, these products are transported down the river course (Bartnik, 1992).

There are many forms of erosion (Graf, 1971). This paper is limited to the process which occur in riverbeds. Bed erosion is a highly natural process which has been observed since ever. It has been intensified by not always proper human activity during last decades. The most important factors are: deforestation which reduces the retention ability of drainage area, hydraulic engineering activity and drawing out coarse material (gravel and cobble).

The aim of the paper is to establish the main trends of changes at the gauging station caused by riverbed erosion in the Carpathian rivers, observed in a long time period. The process of riverbed erosion can be considered as a stochastic process, hence the investigation of the intensity of riverbed erosion presented in this paper is based on the method of statistical analysis. The consideration is based on the minimal and on the mean annual water stages. The assumption is that the minimal annual water stages correspond with the changes of the riverbed level. There is also applied the method of establishing a forecast of the riverbed erosion. The data series analysis leads to establishing the previous trends and the forecast of riverbed erosion in each gauging station. The data series are obtained by hydrological observations which have been carried out in gauging station of a particular river.

The problem presented in this paper is one of the most important in the water engineering. The knowledge of riverbed changes is necessary for future needs and the uses of the river system (designing water intakes, bridges and other water engineering constructions). This knowledge is also connected with the locating of underground water in an adjacent area which is very important for the evaluation of water relations in the soil. The future study of the erosion process and sediment transport is necessary. It should supply more completed data and give the opportunity to prepare a more precise riverbed level forecast.

2. Description of the experimental catchment

2.1. Presentation of Raba River

The Raba river, in southern Poland, is a mountain tributary of the Wisła River. In this region, the topography of drainage catchments is highly varied. The division of the Carpathian rivers courses is as follows: the upper course (with the average slopes: 7 ‰ – 15 ‰), the middle and the lower courses (with the average slopes 2.5 ‰ – 0.5 ‰).

In the Carpathian rivers a high variety of water stages is observed: a rapid-growing flow appears especially in spring and early summer while, in the period of dry season or during the long-lasting snow-cover on rivers, a low-flow period can be observed. In the Raba river, during the flood, discharge can reach the value Q = 800 m^3/s in the upper course, and about Q = 1500 m^3/s in the lower course.

The total area of the Raba River catchment is 1537.10 km^2. In Km 60.00 of River course, Dobczyce dam is located. The Raba River is characterized by erosion and deposition process which occurs with varied intensity along the river course. However, the high erosion process is observed downstream from the Dobczyce Dam (Matyas and Łapuszek, 2000).

2.2. Presentation of the experimental reaches

In the current paper, investigation of riverbed erosion and riverbed evolution is applied for two reach which are located in Gdów (48.3 km of the river course) and in Proszówki (21.7 km of the river course), downstream of Dobczyce Dam. The river in the presented reach was repeatedly straightened and narrowed during the 20th century. The hydraulic engineering activity is the most important factor of high erosion of riverbed in the mentioned reach of the Raba River (Ratomski and Witowska, 1993). The observations show that up to 3 metres of bed degradation has occurred since the beginning of the river training.

3. Method of analysis and computation

The erosion process is evaluated on the base of minimal water stages in gauging stations of Raba River. This investigation can be carried out under the condition that minimal annual water stages correspond to the changes of the riverbed level (Punzet, 1994).In order to prove the above thesis the appriopriate computation in selected gauging stations of the other mountainous tributary (Dunajec River) of Wisla River has been carried out (Łapuszek, 1999).

The results of the computation point out that the relation between height of a water surface above the natural bed in year and time in the period of some years is almost constant in the analysed gauging stations mentioned above. It means that the relation between height of a water surface and time does not change although the changes of the riverbed level in a long-time period can be observed. These analysis lead to the conclusion that the proposed method of evaluation on the base of minimal annual water stages in the controlled gauging stations can be applied to establish the intensity of changes in riverbed and also to forecast the changes.

3.1. Method of computation

The course of low annual water stages in the selected gauging stations has been analysed. In a particular gauging station this data series for each determined time interval is approximated by the linear interpolation. The investigated equation (2) gives the relation between water stage (H) and time (T) and it also illustrates the main trends of changes in the gauging station in a long time period. The equation determining the level of the riverbed at any time is expressed as:

$$H_d = H_z + H_i(T) \qquad (1)$$

where:

H_d - average level of riverbed in a year T [metres above the sea level],
H_z - assumed datum level [metres above the sea level],
T - year of observation,
$H_i(T)$ - approximated equation obtained by:

$$H_i(T) = aT + b \qquad (2)$$

The intensity of changes in time is expressed by the coefficient for T variable.
A few tests have been carried out to divide the data series into time intervals. The final division is the best solution for the least square method of approximation.

In order to confirm the validity of the computation, the functions determining the level of the bed at any time, are verified on the basis of the comparison of:

- the quantity of real changes of the channel cross-section, which are measured in different years,
- the changes of rating curves observed during this period.

On the base of measured amount of the eroded material for the assumed time intervals the real amount is determined. The observations of the changes of rating curves in the selected years can also show the changes in the riverbed level which have been observed during a long time period.
The obtained real values of riverbed erosion have been compared to the suitable coefficients for T variables, which describe the main trends of changes in gauging station in a particular time interval. In the current paper the comparison is presented only for the the quantity of real changes of the channel cross-section, which are measured in different years
On the basis of the obtained equations which describe the riverbed changes, the forecast of the riverbed erosion in controlled station is established.

3.2. Determination of the equations which express the intensity of riverbed erosion

The investigation of the riverbed erosion has been carried out with the assumption that minimal annual water stages correspond with the changes of the riverbed level. Hence, the obtained data series is divided into the time intervals, in which the equation (2) is approximated by the linear interpolation. The mentioned equation (in each ,,i''- time interval) is expressed as:

$$H_i(T) = E(H|T) = \alpha_i T + \beta_i \quad (3)$$

where:

H – low annual water stage [cm],

T – the year of observation.

The assumption is that T and H are dependent variables and they are expressed by a normal bivariate distribution. Coefficients α_i and β_i are the real values of coefficients of the linear regression. In the following description of the computational method the ,,i'' index is omitted.

The determination of the mentioned equation (2) for the bivariate testing data series:$\{(T_j, H_j)\} = \{(T_1, H_1), (T_2, H_2), \ldots (T_n, H_n)\}$ in a particular time interval are performed in the following way (Fisz, 1967):

- estimation of the equation parameters (,,a'' and ,,b''),
- carrying out the test of significance for the coefficient of linear regression,
- establishing the confidence interval for the estimated coefficient of linear regression.

The coefficients of linear regression are established by the least square method:

$$F(a,b) = \sum_{j=1}^{n} [H_j - (aT_j + b)]^2 = \min \quad (4)$$

Introducing the symbols of arithmetic mean $(\overline{T}, \overline{H}, \overline{TH})$, the equations which describe the empirical coefficients of linear regression are as follows:

$$a = \frac{\overline{TH} - \overline{T} \cdot \overline{H}}{T^2 - (\overline{T})^2} \quad (5)$$

$$b = \overline{H} - a\overline{T}. \quad (6)$$

The test of significance $H_0(\alpha=0)$ for coefficient of linear regression '' α'' between the variables (T, H) has been performed on the basis of the assumption that these variables are subordinated by a normal bivariate distribution. In this case, the statistics which is subordinated by two-dimentional Student's distribution, is used for testing previous hypothesis H_0:

$$t^* = \frac{\alpha}{s_R} \sqrt{\sum_{j=1}^{n} (T_j - \overline{T})^2} \quad (7)$$

where:

s_R – deviation to the linear regression:

$$s_R = \sqrt{\frac{1}{n-2} \sum_{j=1}^{n} [H_j - (aT_j - b)]^2} \quad (8)$$

For the significance level γ=0.05 and n-2 degrees of freedom the critical value t_γ is chosen on the basis of the table of two-dimentional Student's distribution. This critical value t_γ is compared to the value of statistics t^*. Satisfied in equation: $|t^*|<t_\gamma$ means that variations H and T are independent. The following satisfied in equation: $|t^*|>t_\gamma$, it means that variations H and T are dependent. The quantity measure of fitting the investigated regression line to the experimental points is obtained by the following formula:

$$S_{RH}^2 = \frac{1}{n} \sum_{j=1}^{n} [H_j - (aT_j + b)]^2 \quad (9)$$

It minimizes the sum of mean square deviation from the regression line (2): $H_i(T)=aT+b$.

This value is the empirical equivalent of the remainder variance: $\frac{(E[\mathrm{var}(H|T)])}{\mathrm{var}\, H}$ hence it gives the precision of the real regression representation by the investigated regression line. The quantity assessment of the obtained regression is given by the confidence intervals. For assessment confidence level 1-γ the confidence interval is established by:

$$a - t_\gamma \frac{s_R}{\sqrt{\sum_{j=1}^{n}(T_j - \overline{T})^2}} < \alpha < a + t_\gamma \frac{s_R}{\sqrt{\sum_{j=1}^{n}(T_j - T)^2}} \tag{10}$$

where:

a – coefficient of regression obtained by the least square method for function (2),

t_γ – variable of t-Student obtained for n-2 degrees of freedom and established confidence coefficient 1-γ.

The quantity assessment of the regression is given for the obtained equation, which are the assessment basis for the riverbed erosion forecast, by the confidence zone. This confidence zone is situated upper and lower curves given by following formula:

$$H_\pm(T) = H(T) \pm t_\gamma(n-2)s_R\sqrt{\frac{1}{n} + \frac{(T_j - \overline{T})^2}{\sum_{j=1}^{n}(T_j - \overline{T})^2}} \tag{11}$$

The computation is carried out for the observed data series in the selected gauging stations. This data series is divided into time intervals in which the mentioned functions are established. The linear regression describes the tendency of riverbed erosion.

3.3. The forecast of riverbed erosion

The estimation of future riverbed erosion is elaborated on the basis of the obtained linear regression for the gauging station. The linear regression is a good tool for the riverbed erosion forecasting for these phenomena for which one can obtain enough data. However, the data must be stable in time.

Hence, the equations mentioned (2) are chosen in order to satisfy these conditions. It means that in the time interval of selected equations the process of riverbed erosion is observed continually but without very rapid changes in the riverbed level. This model should be verified and then it can estimate the riverbed erosion. The forecast is elaborated for the assumed confidence coefficient 1-γ=0.95 according to the following formula (Fisz, 1967):

$$P\{H_p - t_\gamma D(\dot{H}_p) < H_p < \dot{H}_p + t_\gamma D(\dot{H}_p)\} = 1 - \gamma \tag{12}$$

where:

$\dot{H}_p$ - forecast for linear regression given by the formula:

$$\dot{H}_p = aT_p + b \tag{13}$$

where ''a'' and ''b'' are expressed by formulae: (8), (9),

H_p - the estimated future value of low annual water stage [cm],

$D(H_p)$ - mean error of forecast given by formula:

$$D(H_p) = s_R\sqrt{1 + \frac{1}{n} + \frac{(T_p - \overline{T})^2}{\sum_{j=1}^{n}(T_j - \overline{T})^2}} \tag{14}$$

where:

T_p – year.

The length of selected time intervals is different for each gauging station and it depends on the following factors (Łapuszek, 1999, Węglarczyk, 1998):

- length of the data series used as the basis for the forecast,
- value of the mean error,
- possibility of occurring new flow condition connected with water engineering activity in riverbed.

The forecast proposed in the paper is elaborated for a short period, (5-10 years) in the selected gauging station. The flow condition could change during the years, therefore the forecast should be verified every few years.

4. Analysis of the results

The data series of low annual water stages in two gauging stations (Gdów and Proszówki) are divided into time intervals. For each particular interval the linear regression is established and it describes the tendency of riverbed erosion. The proposed division of time intervals is conditioned by the possibility of verification of the obtained results and the observed tendency of the course of annual water stages.

In Gdów gauging station the 96-year period of low annual water stages (1901-1996) has been analysed. This data series is divided into two time intervals: 1901-1952, 1953-1996 (Figure 1). The relation between water stage (H) and time (T) is given by equation H_i=f(T), (i=1,2). It is determined for each time interval (Table 1). The equation determining the level of bed at time T is as follows:

$$H_d = 217.148 + H_i(T) \quad (15)$$

The intensity of changes in time is expressed by the coefficient for T variable. It is easy to remark that the elementary depressions of the bed are adequate to following time intervals: 1.5 cm, and 5.8 cm per year respectively (Table 1).

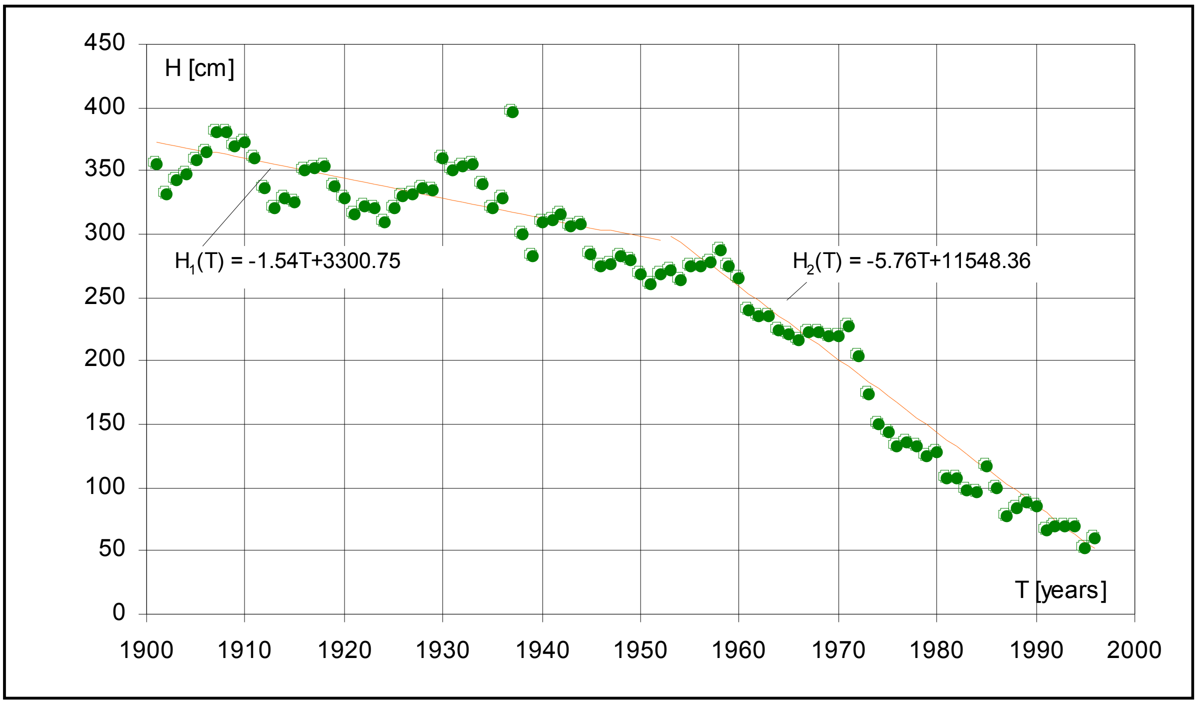

Figure 1. The low annual water stages and linear regressions in Gdów gauging station.

Table 1. Functions determining the riverbed erosion for each time interval in Gdów gauging station.

Time interval	Linear regression	Confidence interval of coefficient α	Correlation coefficient R_{TH}	The most important factors of riverbed erosion
1901-1952	$H_1(T) = -1.54T+3300.75$	$-1.823 < \alpha < -1.257$	-0.00013	- drawing out coarse material (gravel and cobble)
1953-1996	$H_2(T) = -5.76T+11548.36$	$-6.068 < \alpha < -5.452$	-0.00022	- drawing out coarse material (1960-70) - hydraulic engineering activity (narrowing the riverbed), (1960-70) - flood (1970, 1972, 1980)

In Proszówki gauging station the 102-year period of low annual water stages (1901-2002) has been analysed. This data series is divided into three time intervals: 1901-1934, 1935-1982, 1983-2002 (Figure 2). The equation $H_i=f(T)$, $(i=1,2,3)$ is determined for each time interval (Tale 2). The equation determining the level of bed at time T is as follows:

$$H_d = 187.619 + H_i(T)\,. \tag{16}$$

The intensity of changes in time is expressed by the coefficient for T variable. It is easy to remark that the elementary depressions of the bed are adequate to the following time intervals: 0.99 cm, 2.7 cm, 0.6 cm per year respectively (Figure 2).

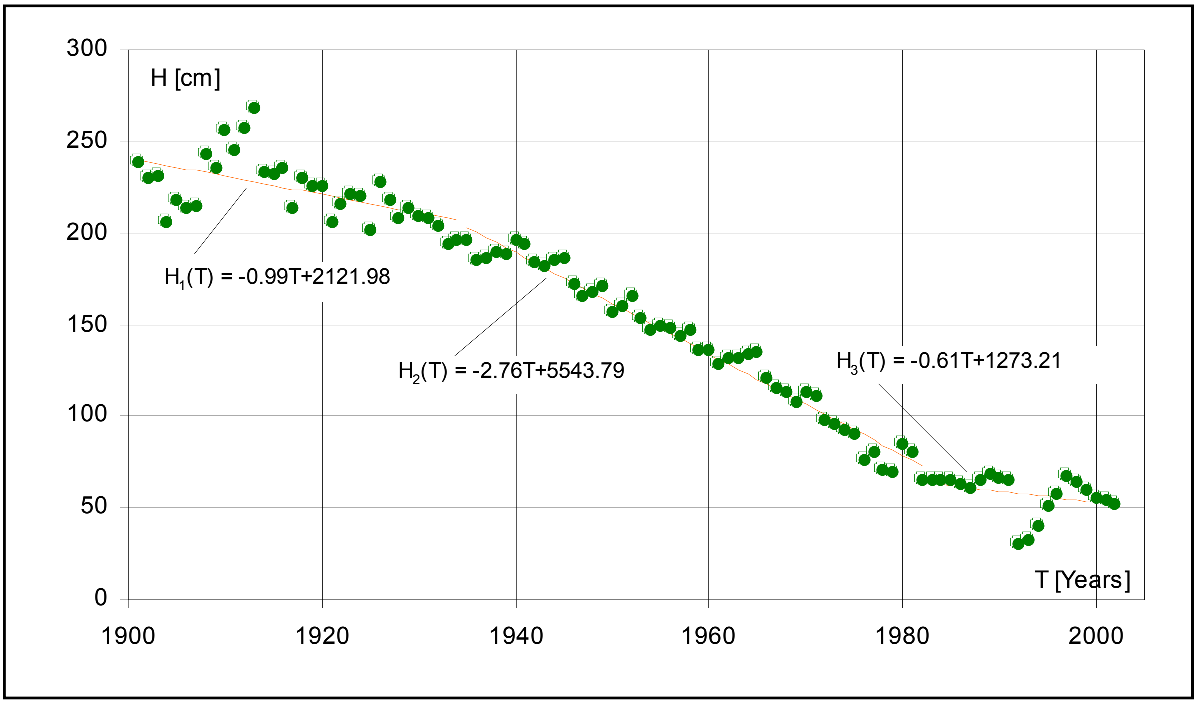

Figure 2. The low annual water stages and linear regressions in Proszówki gauging station.

Table 2. Functions determining the riverbed erosion for each time interval in Proszówki gauging station.

Time interval	Linear regression	Confidence interval of coefficient α	Correlation coefficient R_{TH}	The most important factors of riverbed erosion
1901 - 1934	$H_1(T) = -0.99T+2121.98$	$-1.343 < \alpha < -0.635$	-0.0009	- engineering activity (strightening and narrowing the riverbed), (1920-30)
1935 - 1982	$H_2(T) = -2.76T+5543.79$	$-3.052 < \alpha < -2.468$	-0.0003	- drawing out coarse material (1960-70) - hydraulic engineering activity (1960-70) - floods (1970, 1972, 1980)
1983 - 2002	$H_3(T) = -0.61T+1273.21$	$-1.068 < \alpha < -0.151$	-0.0002	- engineering activity, (1990)

5. Verification of the results

The verification of the established linear regressions in particular gauging station has been carried out on the basis of observed during the years:

- changes of the shape of the channel cross-section (width, depth),
- changes of the discharge curves.

On the basis of the measured quantity of the eroded material for the adequate time intervals the real quantity has been determined. The mean riverbed depression per year is established in a particular time interval. The obtained values in each time interval are compared to the suitable coefficients for T variable of $H_i(T)$.

The verification has been carried out also by the discharge curves analysis. On the basis of the observed changes of the curves in time the mean riverbed depression per year has been established and compared to the suitable coefficients for T variable of $H_i(T)$ in each time interval (Łapuszek, 2003). In the paper, the obtained functions has been verified by the observed changes of the shape of the channel cross-section.

In Gdów gauging station the results of computation are verified by comparing the observed changes of the shape of the channel cross-section in time, in two time intervals (1901-52, 1953-96) (Figure 3). The average riverbed depression is 1.7 cm per year from 1901 to 1952, and from 1953 till 1996 depression is 3,0 cm. The results of verification (Table 3) confirm the correctness of the assumption.

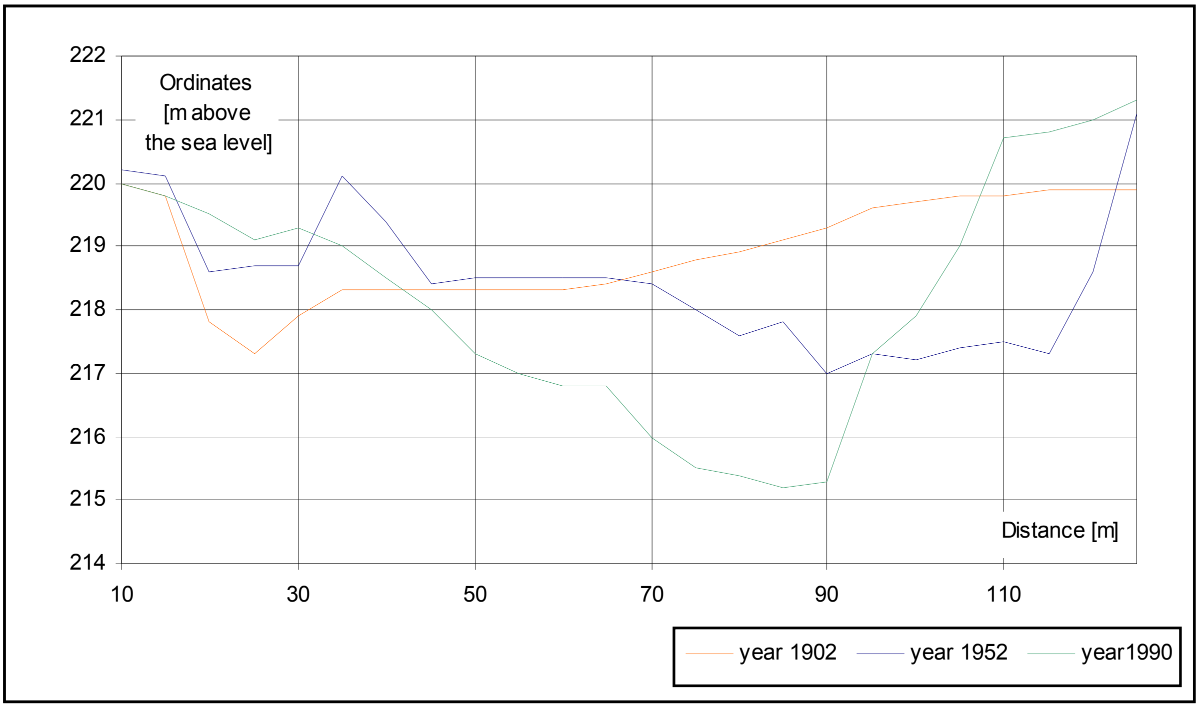

Figure 3. Changes in the shape of the channel cross-section in Gdów gauging station.

Table 3 The intensity of riverbed erosion in Gdów gauging station.

Time interval	*Years*	*Mean riverbed depression obtained by: [cm/year]*	
		Changes of cross-sections	*$H_i(T)$*
1901-1952	52	1.70	1.54
1953-1996	45	3.00	5.76

The equations $H_2(T)$ in Proszówki gauging station are verified on the basis of the observed changes in the shape of the channel cross-section (Figure 4). In this computation the value of mean year depression is 2.6 cm (Table 4).

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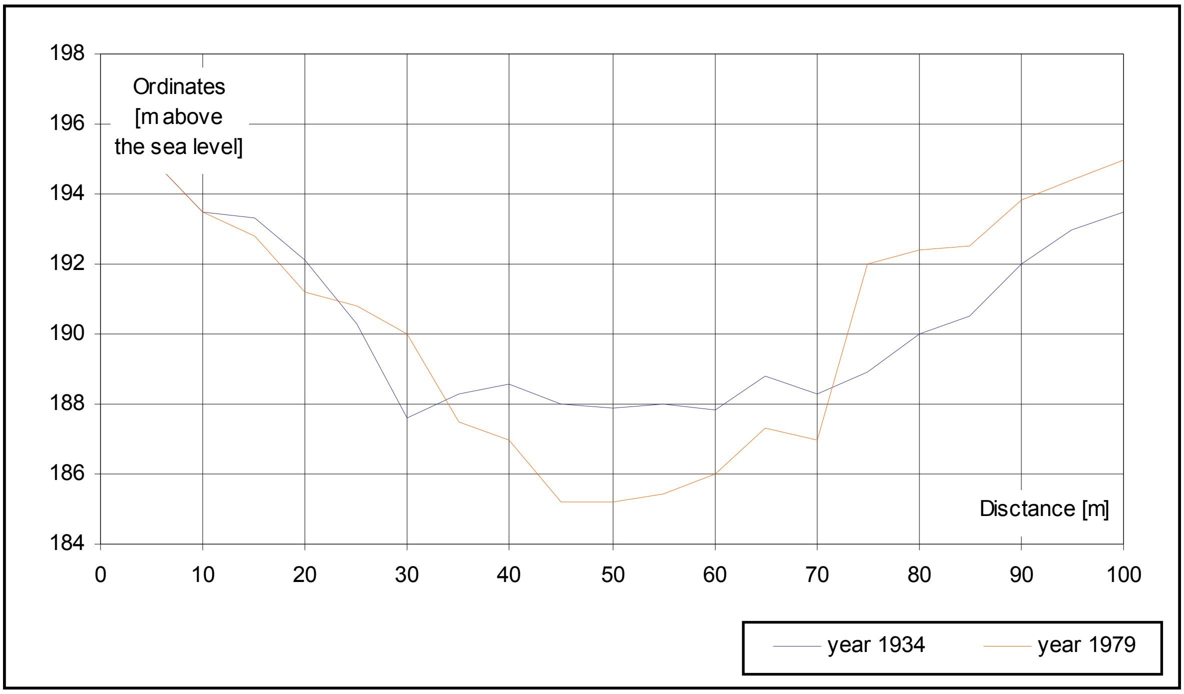

Figure 4. Changes in the shape of the channel cross-section in Proszówki gauging station.

Table 4. The intensity of riverbed erosion in Proszówki gauging station.

Time interval	*Years*	*Mean riverbed depression obtained by: [cm/year]*	
		Changes of cross-sections	*$H_i(T)$*
1901 - 1934	34	No suitable data	0.99
1935 - 1982	49	2.60	2.76
1983 - 2002	21	No suitable data	0.61

In both presented cases the results obtained independently are significantly similar to the value for coefficient of T variable in approximative equations. It confirms the correctness of the assumptions. That is why the method proposed in this paper is suitable for riverbed changing forecast.

6. The riverbed erosion forecasting

According to the assumptions formulated in point 3.3, the forecast of riverbed erosion has been established for Gdów gauging station of the Raba River. The intensity of riverbed erosion for years 1997-2006 is given by the following equation:

$$H_3(T) = -5.76T + 11548.36$$

(17)

The above formula gives the mean erosion value equal 5.7 cm per year. Taking into account also the value of the mean error, the forecast estimated by the performed equation for 10 years (1997-2006) can be proposed and no longer. This forecast should be verified every few years.

The forecast for Proszówki gauging station has not been established yet. It should be performed in a few years, after the riverbed stabilization.

7. Conclusions

1. The presented method of evaluation of mountainous riverbed based on minimal water stages in controlled cross-sections can be applied to establish the intensity of changes in riverbeds.
2. A verification of the established linear regressions in a particular gauging cross-section is carried out independently on the basis of changes of the shape of the channel cross-section. The results of verification confirm the validity of the assumption thus the method proposed in the paper can be used for the riverbed changing forecast.
3. The forecast of riverbed erosion in mountain rivers can be elaborated on the basis of the obtained linear regression. However, the forecast should be verified every few years because of the possibility of some rapid changes of riverbeds connected with water engineering activity or floods.
4. A further study on the erosion process in mountainous rivers is necessary in order to obtain complete data which could permit to prepare a more precise forecast. The knowledge of riverbed changes is important for designing water intakes, bridges and other water engineering constructions. It is also connected to the locating of underground water table in an adjacent area which is very important for the evaluation of water relations in the soil.

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